MONTHLY NOTICES

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ROYAL ASTRONOMICAL SOCIETY.

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W. H. MAW, Esq., PRESIDENT, in the Chair.

Brandon T. Brierley, F.G.S., Assoc.M.Inst.C.E., Linthwaite, Delph, near Oldham, Yorkshire ;
Maurice Farman, Observatoire de Chevreuse, à Jagny, par Dampierre, Seine-et-Oise, France ;
Willie Venner Merrifield, B.A., Liverpool Corporation Nautical College, Liverpool ; and 5 Green Bank, Waterloo, near Liverpool ;
Isaac Molloy, M.A., Lützen, Glenageary, Kingstown, Dublin ;
Alfred Edward Nicholls, King Edward VII. Nautical School, London, E. ; and Cotswold, Hornchurch, Essex ; and
John Wearing, Garsdale, Sedbergh, Yorkshire,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :

Scriven Bolton, 24 Kensington Terrace, Hyde Park, Leeds (proposed by Richard Kerr) ;
Bahne Bonniksen, 16 Norfolk Street, Coventry (proposed by Richard Inwards) ;
Edward Turner Cottingham, Scientific Instrument Maker, The Limes, Thrapston (proposed by Julien Tripplin) ;
William George Hooper, Wiverton House, Musters Road, West Bridgford, Nottingham (proposed by Richard Kerr) ;

Percy Marshall, Librarian, Filey House, Livingstone Road, Scarborough, Yorkshire (proposed by the Rev. T. E. R. Phillips) ; and

Karl Pearson, M.A., LL.B., F.R.S., Professor of Applied Mathematics and Mechanics, University College, London ; 7 Well Road, Hampstead, N.W. (proposed by H. H. Turner).

Ninety presents were announced as having been received since the last ordinary meeting, including, among others :

Six volumes of M.S. Meridian observations made at the Madras Observatory, 1840–51, presented by the India Office ; forty charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich ; stereoscopic views of the stars, presented by T. E. Heath.

A Determination of the Constant of Precession and the Direction of the Solar Motion from a Comparison of Groombridge's Catalogue (1810) *with Modern Greenwich Observations.* By F. W. Dyson, M.A., F.R.S., and W. G. Thackeray.

As a preliminary to the determination of the precession and solar motion an analysis is made of the distribution and proper motions of the stars in terms of star magnitude, size of proper motion, and type of spectrum. Among other results of these analyses the following, which have a direct bearing on the main purpose of the paper, may be cited :

1 (*a*) The two most symmetrical distributions of stars are those of

(*α*) Type II.

(*β*) Total centennial proper motions, between 5″ and 10″.

b. Small proper motions aggregate about 0^h as compared with 12^h.

c. Large proper motions about 18^h are more numerous than might be expected.

2. The values found for the precessional constants are :

$$\text{For 1850 the value of } m = 46''\cdot0757$$
$$n = 20\cdot0531$$

implying an increase to Newcomb's values of $\frac{1}{10000}$th part.

3. In the determination of the solar apex the following systematic discordances occur which cannot be explained by instrumental errors :

a. Stars whose R.A. lie between 6^h–18^h give a smaller value of A than those which lie between 18^h–6^h.

b. Stars of Types I. and II. differ considerably in the positions they assign to the position of the apex of the Sun's motion.

c. The declination of the apex progresses from south to north as the stars' magnitude diminishes, and the right ascension is small for very bright stars, and large for faint stars.

d. Stars whose centennial proper motions are less than 10″ place D north of the position derived from stars whose centennial proper motions are greater than 10″.

4. The most probable value of the position of the apex of the Sun's motion derived from the discussion is

$$A = 275^\circ \text{ or } 18^h\ 20^m$$

$$D = +37^\circ$$

Position of the Sun's Apex from Different Groups of Stars (see p. pp. 445–8).

M.	A.	D.	No. of Stars.	General Remarks.
				General solution.—P.M. 0″–20″.—Zone 5°–52°.
2·40	275°	38°	4001	Equal weights to each star.
2·55	272	31		„ „ equal areas.
				Solutions according to Type of Spectrum.
2·44	269	23	1100	Type I.
2·79	273	37	866	„ II.
2·42	280	41	2035	Remainder
				Equal weight to each star.
				Solutions according to size of P.M.—Zone 5°–52°.
				P.M.
1·17	273	39	2885	0″–5″ Equal weights to each star.
1·08	278	34		0–5 „ „ equal areas
4·47	269	37½	800	5–10 „ „ each star.
8·36	273½	22	316	10–20 „ „ „
25·11	275½	30½	163	> 20 „ „ „
23·5	272½	34	89	Type II. > 20 „ „ „
				Solutions in order of star magnitude.—Zone 15°–52°.
				Mag.
3·96	245	16	200	1·0–4·9
3·18	268	27	454	5·0–5·9
2·85	278	33	1003	6·0–6·9
2·24	280	38½	1239	7·0–7·9
2·32	272	43	811	8·0–8·9
				Equal weights to equal areas.

I. *Introductory.*

The principal object of the Greenwich Second Ten-Year Catalogue (1890) was the reobservation of the stars of Groombridge's well known Circumpolar Catalogue, which had not been observed at Greenwich as recently as the period of the Nine-Year (1872) Catalogue. Of the 4239 stars contained in Groombridge's Catalogue 3645 occur in the Second Ten-Year (1890) Catalogue, and most of the remainder in the Ten-Year (1880) Catalogue. The small number of Groombridge stars not occurring in these catalogues are found in the Nine-Year (1872) Catalogue, or have been observed since the conclusion of the 1890 Catalogue. There has thus been accumulated material for the determination of the proper motions of a large number of stars with an average interval between the observations of nearly eighty years.

Groombridge's Catalogue has long been known to have somewhat large systematic errors, and from the remarks made by Sir George Airy in the introduction it seemed that there might be some errors of computation which an examination of the original observations might discover. Accordingly the loan of the Groombridge manuscripts was requested by the Astronomer Royal from the Royal Astronomical Society, and a complete re-reduction of the observations made. This re-reduction is now completed, and proper motions have been deduced by comparison of the revised positions with the modern Greenwich positions.

The observations on which Groombridge's Catalogue was based were made by him at Blackheath between the years 1806 and 1816 with a reversible Transit Circle made by Troughton. The original observations were apparently entered on a slate by Groombridge, and copied out the next morning into two books, the right ascensions in one book and the north polar distances in the other. The times of transit across the separate wires were entered as originally observed, but the readings of the separate microscopes were first converted into readings of zenith distance, and these two sets of entries have formed the basis of the new reduction, and none of the original computations have been utilised. The results of the re-reduction bear out Sir George Airy's remark that the instrument was one of the best of its time, and was well handled by Groombridge.

The re-reduction of the observations has been carried out in such a way as to preserve as far as possible their fundamental character. The positions of the clock stars, azimuth stars, and stars for polar point have been derived from Newcomb's Fundamental Catalogue. The main difficulties of the reduction have consisted in the absence of material in the right ascensions for separating the collimation error from the azimuth error. A correction of the form $m + n \tan \delta$ has been applied to the observed times of transit, and not one of the form $m + n \tan \delta + c \sec \delta$, and as far as possible use has been made of reference stars of the same declination as that of the majority of the stars under observation.

In the declinations the difficulty of the determination of the division errors appears to have been satisfactorily overcome by comparison of the two positions of the instrument with Newcomb's Fundamental Catalogue, and by combining the results obtained for zenith distance north in one position with those of an equal distance south in the reversed position of the instrument. Thus corrections were found applicable from 39° to 52° N.P.D., and from 38° to 25°, or from 25° to 52° in all. The corrections from 10° to 25° N.P.D. were obtained from each position singly, and from 0° to 10° the material was so insufficient that no corrections have been applied. As the reduced catalogue is now in the press it is unnecessary to give further details of the reductions.

The Greenwich catalogues have been corrected to reduce them to Newcomb's Equinox in R.A., and to the Pulkowa refractions in N.P.D., in order that they may be strictly comparable with Groombridge's Catalogue. In the determination of the proper motions the Struve-Peters values of the constants m and n were used. The precessions were computed for 1810 and 1890, and the mean used in bringing the observations from 1810 to 1890, with a correction for third term where necessary. The deduced proper motions are therefore referred to the epoch 1850. In a small number of cases the observations of Groombridge were supplemented by those of the Radcliffe Catalogue, 1845, and very occasionally the position given by Groombridge was rejected, and the proper motions derived entirely from the Radcliffe and Greenwich Catalogues.

These observations of proper motion are in some ways very suitable for a determination of the constant of precession and of the solar motion. The stars are of mean magnitude $7^{m}{\cdot}0$, and therefore such as might well have been observed without any large errors depending on magnitude, and on the other hand they do not consist of too exclusively bright stars. The constant of precession and direction of the solar motion have been obtained from the proper motions of Auwers' Bradley by several distinguished astronomers, and therefore a determination made from the proper motions of the stars used by Groombridge is desirable, as it consists largely of fresh material, which, however, is confined to a very limited region of the sky. The proper motions in right ascension have from the outset been converted into arc.

II. *Distribution of the Stars according to* (i.) *Magnitude,* (ii.) *Size of Proper Motion, and* (iii.) *Type of Spectrum.*

(i.) *Distribution of Stars according to Magnitude.*

The stars observed in Groombridge's Catalogue lie within 52° of the North Pole, and are, generally speaking, the brighter stars in this part of the sky. It is only very occasionally that a star as faint as $9^{m}{\cdot}0$ is observed, and the mean magnitude is exactly $7^{m}{\cdot}0$. The magnitudes are taken from the *Harvard*

Durchmusterung where possible, and in other cases from the catalogues of the *Astronomische Gesellschaft* or the *Bonn Durchmusterung*. The following table is an analysis of the stars' magnitudes in each octant of right ascension :—

Limits of R.A. h h	Var.	$1^{m}.0$–$4^{m}.9$.	$5^{m}.0$–$5^{m}.9$.	$6^{m}.0$–$6^{m}.9$.	$7^{m}.0$–$7^{m}.9$.	$8^{m}.0$–$8^{m}.9$.	$9^{m}.0$–$9^{m}.5$.	Total.	Mean Mag. m
0–3	3	34	77	148	204	151	5	622	7·1
3–6	1	37	67	139	126	109	4	483	6·9
6–9	...	19	60	124	140	70	2	415	6·9
9–12	...	22	51	102	111	45	1	332	6·8
12–15	...	25	57	109	85	52	7	335	6·7
15–18	...	27	51	105	102	36	0	321	6·7
18–21	3	35	78	216	306	238	8	884	7·2
21–0	2	35	97	206	295	206	6	847	7·1
Total ...	9	234	538	1149	1369	907	33	4239	7·0

It will be seen from the above table that only one-third of the stars fall between 6^{h} and 18^{h}. This peculiarity in the distribution of the stars is in no way attributable to Groombridge's habits of observing, but is entirely due to the actual differences of star-density in different parts of the area covered by the catalogue. Comparison with Seeliger's counts of the total number of stars of different magnitudes shows that Groombridge observed the same percentage of stars of the same magnitude throughout. In the portion of the sky observed by him his catalogue contains :

			m		m
	70 per cent.	of the stars from	1·0	to	6·5.
	35	„ „	6·6	„	7·0.
	15	„ „	7·0	„	7·9.
and	3	„ „	8·0	„	8·9.

The large differences in the number of stars in different parts of the sky, suggesting the probability of systematic differences in the mean distances of groups of stars in different parts of the sky, make it desirable in some cases to make two solutions for the direction of the solar motion, one giving the same weight to equal areas of the sky, and the other to each star. As will be seen in Section V., when the weight is according to area the position of the solar apex is some degrees further south than when the weight is according to the number of stars.

(ii.) *Distribution of Stars according to Size of Proper Motion.*

The stars were divided into groups according as the total proper motion per century fell between the limits 0″ to 5″, 5″ to 10″, 10″ to 20″, or was greater than 20″. The following table gives the numbers and percentage of stars belonging to these groups for different magnitudes :—

Mag.	Total Number.	Numbers of Stars. Centennial Proper Motion.				Percentage of Stars. Centennial Proper Motion.			
m m		0″–5″.	5″–10″.	10″–20″.	>20″.	0″–5″.	5″–10″.	10″–20″.	>20″.
1 –3·9	71	20	15	23	13	28	21	32½	18½
4·0–4·9	163	86	34	23	21	53	21	14	12
5·0–5·9	538	319	113	73	33	59	21	14	6
6·0–6·9	1149	764	239	95	51	66½	21	8	4½
7·0–7·9	1369	1008	248	78	36	74	18	5½	2½
8·0–	940	738	159	24	19	78½	17	2½	2
	4230	2935	808	316	173	69	19	8	4

An interesting feature in the above table is the consistency in the proportion of stars whose proper motion lies between 5″–10″ a century.

For the question of the solar motion it is most important to see how the proper motions are distributed in the sky. This is shown in the following tables, where the number of stars and percentages in these various groups are given for each octant :—

*Numbers of Stars whose Total Proper Motions lie between Certain Limits.**

Limits of R.A.	Total Number.	6ᵐ·5 and Brighter. Centennial Proper Motion.				Total Number.	6ᵐ·6 and Fainter. Centennial Proper Motion.			
h h		0″–5″.	5″–10″.	10″–20″.	>20″.		0″–5″.	5″–10″.	10″–20″.	>20″.
0– 3	188	109	44	21	14	427	334	67	19	7
3– 6	179	127	31	16	5	301	215	61	15	10
6– 9	147	80	38	21	8	267	185	58	21	3
9–12	124	50	37	26	11	205	115	52	24	14
12–15	149	66	31	35	17	181	120	40	14	7
15–18	129	66	32	15	16	192	130	38	15	9
18–21	234	146	43	26	19	641	513	96	18	14
21– 0	232	160	37	24	11	611	484	103	16	8
	1382	804	293	184	101	2825	2096	515	142	72

Percentage of Stars whose Total Proper Motions lie between Certain Limits.

Limits of R.A.	6ᵐ·5 and Brighter. Centennial Proper Motion.				6ᵐ·6 and Fainter. Centennial Proper Motion.			
h h	0″–5″.	5″–10″.	10″–20″.	>20″.	0″–5″.	5″–10″.	10″–20″.	>20″.
0– 3	59	22	11½	7½	78	16	14½	1½
3– 6	71	17½	8½	3	71½	20	5	3½
6– 9	55	25	14½	5½	69½	22	7½	1
9–12	41	30	20	9	56	25½	11½	7
12–15	44	20	24½	11½	66	22½	7½	4
15–18	51	25	11½	12½	67	20	8	5
18–21	63	18	11	8	80	15	3	2
21– 0	69	16	10½	4½	79½	16½	2½	1½
	58½	21	13	7½	74	18	5	3

* It will be noticed that these numbers do not exactly tally with those given above. A few stars have probably been omitted accidentally in the second table.

The most noticeable features in the above tables are :—

(i.) Stars with large proper motions seem to be more uniformly distributed than those with small proper motions, the latter showing a large increase about 0^h as compared with 12^h.

(ii.) Nearly 60 per cent. of the bright stars have a proper motion of less than $5''$ a century, and more than 25 per cent. of the faint stars have a proper motion of more than $5''$ a century.

(iii.) The tables of percentages show the large relative preponderance of stars of large proper motion—about 12^h of right ascension. Taking the proper motions as a gauge of the distances of the stars, there is here shown a want of symmetry in the distribution of the stars which may well affect the determination of the position of the solar apex.

(iii.) *Distribution of the Stars and their Proper Motions according to Type of Spectra.*

It has been shown by Kapteyn, from a discussion of the proper motions of the Bradley stars, that in general stars of Type II. have larger proper motions than those of Type I. It will be seen that this result is borne out by the following analysis of the stars common to the Groombridge and the Draper Catalogue (*Harvard Annals*, vol. xxvii.) under the headings of Type I. (classified by Professor Pickering as A, B, C, or D) and of Type II. (classified by the letters E to L). There are altogether 2096 Groombridge stars in the Draper Catalogue, distributed as follows :—

Distribution of Proper Motions of Stars of Type I.

Limits of R.A.	Mean Mag.	Total Number.	Numbers of Stars. Centennial Proper Motion.				Percentage of Stars. Centennial Proper Motions.			
h h	m		0″–5″.	5″–10″.	10″–20″.	>20″.	0″–5″.	5″–10″.	10″–20″.	>20″.
0– 3	6·6	211	154	43	11	3	73	20	5½	1½
3– 6	6·3	174	131	32	9	2	75	19	5	1
6– 9	6·4	105	76	24	4	1	72	23	4	1
9–12	6·2	72	41	21	6	4	56	30	8	6
12–15	5·7	85	46	15	20	4	53	18	24	5
15–18	6·0	92	64	20	6	2	70	22	6	2
18–21	6·2	162	134	19	6	3	80	12	4	2
21– 0	6·3	221	176	31	10	4	79	14	5	2
	6·3	1121	822	205	72	23	73	18	7	2

Distribution of Proper Motions of Stars of Type II.

Limits of R.A.	Mean Mag.	Total Number.	Numbers of Stars. Centennial Proper Motions.				Percentage of Stars. Centennial Proper Motions.			
h h	m		0″–5″.	5″–10″.	10″–20″.	>20″.	0″–5″.	5″–10″.	10″–20″.	>20″
0– 3	6·1	132	70	33	15	14	53	25	11	10
3– 6	6·0	106	68	16	15	7	64	15	14	7
6– 9	6·3	136	67	39	22	8	49	29	11	6
9–12	6·1	115	46	31	24	14	40	27	21	12
12–15	6·2	111	51	26	19	15	46	23	17	14
15–18	6·3	114	60	26	12	16	53	23	10	14
18–21	6·2	136	68	33	21	14	50	24	16	10
21–24	6·1	124	70	30	15	9	57	24	12	7
	6·2	974	500	234	133	97	51½	24	14⅓	10

Comparison of the above tables shows an entire difference in the distribution of the stars of Types I. and II. The stars of Type II. are distributed with extreme regularity in the different octants. On the other hand, the stars of Type I. have a very marked condensation between 18^h and 6^h where the Milky Way passes through the observed area. This is especially the case with the stars of smallest proper motion ($0''$–$5''$), and it can also be seen in the stars whose proper motions are between $5''$ and $10''$. The larger the proper motion of Type I. stars the more equal is the distribution in the octants.

Since the above tables were prepared vol. lvi. No. 1 of the *Annals of the Harvard College Observatory* has been received in which Professor Pickering gives an analysis of the distribution of the stars of different types in the whole of the Draper Catalogue, and concludes as follows: "The Universe is thus shown to consist of two portions, first, the stars of the first type, which though frequent in all parts of the sky predominate along a certain plane, thus forming the Milky Way. . . . The stars of the second and third types show no concentration in the Milky Way, but are, in general, uniformly distributed in all parts of the sky. These two portions should be treated separately in all discussion of the structure of the Universe, such as studies of proper motion, parallax, motion of the Sun in space, &c." The tables given above entirely confirm Professor Pickering's statements, and extend them in that they show that the uniformity of distribution of Type II. stars holds good when the stars are subdivided according to the magnitude of their proper motions.

The relatively large proper motions of stars of Type II. have been pointed out by Kapteyn from a discussion largely of Bradley proper motions; in brief the larger the proper motion the greater the chance that it is a star of Type II. This peculiarity is borne out by the following comparison of the present results with those given by Kapteyn. It should be noted that the lists are not entirely independent, as 627 of the 2095 Groombridge stars (*i.e.* 30 per cent.) are also Bradley stars.

Statistical Distribution of Proper Motions of Stars of Types I. and II.

Centennial Proper Motions.	Number of Stars. (Groombridge).		Numbers given by Kapteyn (Bradley).	
" "	Type I.	Type II.	Type I.	Type II.
0– 5	822	500	786	474
5–10	205	234	203	194
10–20	72	133	159	223
20–50	19	77	38	157
>50	4	20	3	58

The tables given above point very clearly to the special suitability of stars of Type II. for researches on the direction of the

solar motion. Their distribution in space appears to be the same in all directions from the Sun ; the proper motions show their mean distance from the Sun, and distribution with respect to this mean to be the same in all directions ; and, further, their distances are such that the effect of parallactic motion is considerable. The stars of Type I. are condensed near the Milky Way, and this is especially the case with the stars of small proper motion. It would seem that the mean distances of these stars are different in different directions, and a difficulty is at once introduced into the determination of the solar motion from them.

III. *The Precessional Constant* m.

In this section the mean proper motions are given for each three hours of right ascension, and the material is arranged for discussion of (*a*) the magnitude equation of Groombridge as compared with the modern Greenwich observers, and the possibility of a rotation of the bright relatively to the faint stars, (*b*) a determination of the precessional constant *m*. The advantages of an arrangement in groups of three hours are that the data are compressed into small compass, and their general character more readily perceived than in more extended tables.

(*a*) *Mean Proper Motions of Stars of Different Magnitudes for each Octant.*—The proper motions in right ascension having been first converted into arc of a great circle, the means were taken for all the stars between the limits $15° - 52°$ of N.P.D. for each three hours of right ascension. No stars were omitted, and the results are shown in the following table :—

Mean Centennial Proper Motions of all Stars between the Limits $15° - 52°$ *N.P.D.*

	Right Ascension.					North Polar Distance.				
Limits of R.A.	1^m– $4^m{\cdot}9$.	$5^m{\cdot}0$– $5^m{\cdot}9$.	$6^m{\cdot}0$– $6^m\,9$.	$7^m{\cdot}0$– $7^m{\cdot}9$.	$8^m{\cdot}0$– $8^m\,9$.	1^m– $4^m{\cdot}9$.	$5^m{\cdot}0$– $5^m{\cdot}9$.	$6^m{\cdot}0$– $6^m{\cdot}9$.	$7^m\,0$– $7^m{\cdot}9$.	$8^m{\cdot}0$– $8^m{\cdot}9$
h h	″	″	″	″	″	″	″	″	″	″
0– 3	+10·5	+8·2	+2·1	+2·5	+3·0	+4·6	+0·5	+1·9	+1·4	+1·
3– 6	+ 1·8	+0·8	+1·3	+1·5	+0·7	+5·5	+3·3	+2·9	+3·3	+2·
6– 9	− 8·3	−2·0	−1·1	−1·0	−1·8	+6·4	+2·5	+3·6	+2·5	+2·
9–12	−10·5	−5·3	−1·1	−2·7	−4·4	+3·8	+1·9	+9·5	+1·1	+3·
12–15	− 8·1	−4·9	−3·3	−2·8	−3·3	−1·8	−0·2	0·0	−0·3	+0·
15–18	+ 1·1	+0·3	−2·2	−2·4	−0·1	−6·3	0·0	+0·2	−0·2	+0·
18–21	+ 5 3	+1·2	+0·6	+0·5	−0·1	+1·5	−0·9	0·0	−0·4	−0·
21–0	+ 5·6	+3·4	+2·0	+1·2	+0·5	+2·7	0·0	+0·4	+0·7	+0·
Mean	− 0·33	+0·21	−0·21	−0·40	−0·69	+2·05	+0·89	+2·31	+1·01	+1

The large accidental quantities (*e.g.* 8″·2 in column 3 and 9″·5 in column 9) are due to a few very large proper motions. The tables were accordingly re-formed, omitting all stars whose total centennial proper motion was greater than 20″.

Mean Centennial Proper Motions of Stars between the Limits 15°–52° *N.P.D.* (*omitting Stars whose Proper Motions are* > 20″).

Limits of R.A. h h	Right Ascension. 1m–4m·9.	5m·0–5m·9.	6m·0–6m·9.	7m·0–7m·9.	8m·0–8m·9.	North Polar Distance. 1m–4m·9.	5m·0–5m·9.	6m·0–6m·9.	7m·0–7m·9.	8m·0–8m·9.
	″	″	″	″	″	″	″	″	″	″
0– 3	+0·6	+0·5	+1·8	+1·5	+0·9	+1·9	+1·8	+1·5	+1·4	+1·1
3– 6	+0·2	+0·8	+1·1	+1·0	+0·7	+2·6	+3·3	+2·3	+2·1	+2·0
6– 9	−3·4	−1·8	−1·3	−1·0	−1·8	+4·1	+2·1	+2·9	+2·2	+2·2
9–12	−4·6	−3·3	−2·7	−2·1	−3·1	+0·5	+1·8	+1·2	+0·7	+1·3
12–15	−1·0	−4·3	−2·6	−1·7	−1·7	−1·5	−0·6	−0·2	−0·1	−0·1
15–18	+0·3	−0·6	−1·6	−1·5	−0·1	−2·1	−1·7	−0·7	0·0	+0·1
18–21	+2·2	+0·7	+0·5	+0·4	−0·1	−3·3	+0·5	+0·3	−0·1	+0·1
21– 0	+3·2	+2·8	+1·5	+1·1	+0·8	+2·0	−0·4	+0·6	+0·7	+0·5
Mean	−0·31	−0·65	−0·41	−0·29	−0·55	+0·52	+0·85	+0·99	+0·86	+0·90

The solar motion is shown very clearly in each column of these tables, and will be fully considered in section V. Taking the results of the latter table as being more free from accidentally large proper motions, the means in R.A. are as follows :—

	m m	″
Mags.	1·0–4·9	−0·31
,,	5·0–5·9	−0·65
,,	6·0–6·9	−0·41
,,	7·0–7·9	−0·29
,,	8·0–8·9	−0·55

By taking the simple means the effect of solar motion is eliminated. The agreement of the results for all magnitudes shows that there is no rotation as a whole of the bright stars relatively to the faint stars in this part of the sky, and that Groombridge's magnitude equation was substantially the same as that of the modern Greenwich observers. The general mean of the R.A. results may arise from systematic errors, erroneous precession, or an accumulation of large proper motions about 12h. Detailed consideration of these points follows. The mean results in N.P.D. are directly attributable to the solar motion.

(*b*) *The Precessional Constant* m.—The agreement of the mean proper motions in right ascension for stars of different magnitudes suggests that this method may be used for a determination of the precessional constant *m*. Omitting stars of centennial proper motion > 20″, and all stars within 15° of the pole, we

obtain the following table of mean proper motions in zones of N.P.D. for each three hours of right ascension :—

Mean Proper Motion in Right Ascension for all Stars whose Centennial p.m. < 20″.

h h	15°–25°.	25°–35°.	35°–45°.	45°–52°.
	″	″	″	″
0– 3	+2·32	+1·79	+0·94	+0·81
3– 6	+1·82	+0·53	+0·97	+0·32
6– 9	−0·82	−0·72	−1·99	−1·78
9–12	−1·65	−2·56	−1·86	−4·20
12–15	−1·78	−1·71	−2·36	−3·54
15–18	−0·34	−0·68	−1·16	−1·57
18–21	+1·21	+1·02	+0·65	−0·08
21–24	+2·24	+1·71	+1·30	+0·91
Mean ...	+0·38	−0·05	−0·44	−1·14

The large value of the mean −1″·14 for the zone 45°–52° calls for some examination. The question arises whether it is due to systematic error in the Groombridge Catalogue at this N.P.D. or whether it may be attributed to those inequalities in the distribution of the stars and their proper motions to which attention was drawn in the last section. At 0^h this zone is very near the Milky Way, and at 12^h a long way from it. Examination shows that the mean proper motions in R.A. arising from stars whose total proper motions lie between certain limits in this zone are divided as follows :—

Analysis of Mean Proper Motions in R.A. for Zone 45°–52°.

R.A. h h	0″–5″.		5″–10″.		10″–20″.	
		″		″		″
0– 3	94	−0·01	23	+2·10	16	+ 3·77
3– 6	76	+0·01	33	+0·24	4	+ 2·83
6– 9	80	−0·85	31	−3·09	16	− 5·12
9–12	40	−1·32	35	−3·87	22	− 9·96
12–15	50	−1·10	24	−4·63	15	−10·75
15–18	46	−0·33	21	−1·38	10	− 7·40
18–21	294	−0·11	58	−0·18	12	+ 1·53
21– 0	160	+0·01	30	+3·66	7	+ 9·73

This table shows the irregularity in the distribution of the stars of large and small proper motion. Thus, between 21^h and 3^h, where the solar motion gives positive motion in right ascension, the number of stars are in the ratio

$$77 : 16 : 7$$

while between 9^h and 15^h, where the solar motion gives negative motion in right ascension, these numbers are in the ratio

$$48 : 32 : 20$$

The large negative proper motions at 12^h have a very con-

siderable effect on the mean, and tables corresponding to the one on p. 438 have been formed in which more stars have been excluded. The two limits chosen are 10″ a century and 5″ a century. The former appears to be a sufficiently ample limit, but the latter is probably too small.

Mean Proper Motion in Right Ascension for all Stars whose Centennial p.m. < 10″.

	15°–25°.	25°–35°.	35°–45°.	45°–52°
0– 3	+1″·85	+1″·78	+0″·60	+0″·40
3– 6	+1·55	+0·31	+0·50	+0·08
6– 9	−0·87	−0·64	−1·66	−1·49
9–12	−1·48	−1·63	−1·71	−2·51
12–15	−0·24	−1·86	−1·31	−2·14
15–18	−0·42	−0·65	−1·15	−0·70
18–21	+0·81	+0·51	+0·51	−0·12
21–24	+1·69	+1·31	+0·93	+0·55
Mean ...	+0·36	−0·11	−0·41	−0·73

Mean Proper Motions in Right Ascension for all Stars whose Centennial p.m. < 5″.

	15°–25°.	25°–35°.	35°–45°.	45°–52°.
0– 3	+1″·13	+1″·32	+0″·43	−0″·01
3– 6	+1·27	+0·13	+0·38	+0·01
6– 9	−0·46	−0·63	−0·95	−0·85
9–12	−0·75	−0·66	−0·70	−1·32
12–15	−0·63	−1·36	−0·80	−1·10
15–18	−0·37	−0·67	−0·85	−0·33
18–21	+1·07	+0·37	+0·33	−0·11
21–24	+1·07	+0·77	+0·68	+0·01
Mean ...	+0·29	−0·09	−0·19	−0·46

Collecting the results we find for the mean proper motions in the four zones :

Limits of p.m.	15°–25°.	25°–35°.	35°–45°.	45°–52°.
0″–20″	+0″·38	−0″·05	−0″·44	−1″·14
0–10	+0·36	−0·11	−0·41	−0·73
0– 5	+0·29	−0·09	−0·19	−0·46
Adopted values	+0·30	−0·10	−0·30	−0·60

These adopted values, when corrected for the systematic errors of the catalogue, form corrections $\Delta m \cos \delta_0$ to the adopted centennial value of the precessional constant m, δ_0 being the mean declination for each group.

The following table gives the corrections to Groombridge's Catalogue to reduce it to the systems of Newcomb, Auwers, and Boss. The comparison with Newcomb has been made directly, that with Auwers is derived from his recently published tables * (the results of the new reduction having been sent to him by the Astronomer-Royal in advance), and that with Boss by applying the corrections Boss-Newcomb for 1810, derived from *Ast. Journ.* 531–2 to the corrections Newcomb-Groombridge.

N.P.D.	Newc.-Groomb.	Auwers-Groomb.	Boss-Groomb.	Boss-Newc
°	s	s	s	s
10	+·18	+·21	+·09	−·09
18	+·14	+·14	+·13	−·01
20	+·06	+·06	+·06	·00
25	+·04	−·03	+·03	−·01
30	−·02	−·03	−·01	+·01
35	−·06	−·03	−·04	+·02
40	−·05	−·04	−·03	+·02
45	−·05	−·06	−·02	+·03
50	−·04	−·08	−·02	+·02

As Newcomb's catalogue was used in the reductions, and as corrections to the equinoxes of the Greenwich catalogues were applied to bring them into agreement with this system, we shall adopt the corrections given by the direct comparison with Newcomb as follows :—

Zone	Corr. to Groombridge.	Corr. to Centennial p.m.
° °	s = "	"
45–52	−·042 = −0·47	+·59
35–45	−·052 = −0·50	+·63
25–35	−·015 = −0·11	+·14
15–25	+·075 = +0·38	−·48

Applying these systematic corrections to the adopted values of the centennial mean proper motion of the different zones given above, we obtain the following equations for Δm :—

Zone ° 45–52	·75Δm = −" ·01	Wt. 7
„ 35–45	·64Δm = +·33	„ 8
„ 25–35	·50Δm = +·04	„ 4
„ 15–25	·34Δm = −·18	„ 3

* *Tafeln zur Reduction von Sterncatalogen auf das System des Fundamentalcatalogs des Berliner Jahrbuchs.* Kiel, 1904.

The solution of the above equations gives $\Delta m = +''\cdot 18$ as the centennial correction to the Struve-Peters value of m, or $+''\cdot 70$ to Newcomb's value. If the right ascensions of Groombridge had been reduced to Auwers's system by the corrections given by him, the deduced value of m would have been about $0''\cdot 2$ per century larger, and reduced to Boss's system about $0''\cdot 4$ smaller.

IV. *The Precessional Constant* n.

To determine the value of the precession, we should, if we could pick them out, use stars so distant that they are unaffected by the solar motion. Let X, Y, Z be the centennial displacement of the Sun relative to any class of stars, and let Δn be the correction required by the precessional constant n. Then m will require a correction, $\Delta n \cot \varepsilon$, where ε is the obliquity of the ecliptic ; and each star will give rise to two equations :

$$X \sin \alpha - Y \cos \alpha + \Delta n(\cot \varepsilon \cos \delta + \sin \alpha \sin \delta) = \mu \quad \text{(i.)}$$

$$\text{and } X \cos \alpha \sin \delta + Y \sin \alpha \sin \delta - Z \cos \delta + \Delta n \cos \alpha = -\mu' \quad \text{(ii.)}$$

If the stars were so distant that the displacement of the Sun relative to them were zero, X, Y, and Z would vanish, and the above equations would give Δn. Apart from errors of observation μ and μ' would be entirely resolved into corrections to the adopted precession. If, on the other hand, we are dealing with a class of stars near the Sun, the displacements X, Y, Z will be large, and only a trifling part of the proper motions μ and μ' will be due to error of precession.

In this section the stars are divided into groups according to the magnitude of their proper motion, and the value of the precessional constant n is deduced from the consideration that stars of small and large proper motion should give the same value of the ratio X : Y.

The limitation of Groombridge's observations to within 52° of the pole would make the normal equations derivable directly from equation (ii.) unsatisfactory, and, though not quite to the same extent, the equations derivable from (i.). In both cases X and Δn would be difficult to separate.

Instead of treating Δn as an unknown quantity to be determined directly, equations have been formed giving the coordinates of the apex of the solar motions for stars with proper motion (1) between $0''$ and $5''$; (2) between $5''$ and $10''$; (3) between $10''$ and $20''$; and (4) $> 20''$ on two suppositions :

First, basing the proper motions on the centennial value of the Struve-Peters constants of precession, viz.

$$m = 4607''\cdot 63 \qquad n = 2005''\cdot 64$$

Second, basing the proper motions on the centennial value of Newcomb's constants of precession, viz.

$$m = 4607''\cdot11 \qquad n = 2005''\cdot11$$

This latter object was affected by applying a correction to the centennial proper motions of $+0''\cdot52 \cos\delta + 0''\cdot53 \sin\delta \sin\alpha$ to the right ascensions and $+0''\cdot53 \cos\alpha$ to the declinations. Roughly speaking, this change in the precession adds $+0''\cdot40$ to the value of X derived from the right ascensions and $+0''\cdot65$ to the value derived from the declinations. The actual values found for X and Y are as follows :—

Limits of p.m.	Right Ascensions.				Declinations.			
	Struve-Peters.		Newcomb.		Struve-Peters.		Newcomb.	
	X	Y	X	Y	X	Y	X	Y
$>20''$	$+2''\cdot87$	$-21''\cdot83$	$+3''\cdot25$	$-21''\cdot85$	$+''\cdot28$	$-20''\cdot88$	$+''\cdot92$	$-20''\cdot89$
10–20	+ ·54	− 7·62	+ ·95	− 7·65	−·60	− 8·54	+·11	− 8·54
5–10	− ·29	− 3·68	+ ·09	− 3·74	−·50	− 3·46	+·14	− 3·46
0–5	− ·10	− 1·02	+ ·25	− 1·22	−·43	− ·70	+·20	− ·70
0–5	− ·05	− 0·92	+ ·38	− 0·92	−·42	− ·81	+·20	− ·81

Dividing X by Y to get the R.A. of the apex of the Sun's motion $\left(\frac{X}{Y} = \cot A\right)$ the divergence in the values of A for the smaller proper motions is shown in the following table :—

Right Ascension of the Direction of the Sun's Motion from Stars whose total Proper Motion lies between certain Limits.

Centennial Proper Motion.	Right Ascensions.		Declinations.		Combined Results.	
	Struve-Peters' Precession.	Newcomb's Precession.	Struve-Peters' Precession.	Newcomb's Precession.	Struve-Peters' Precession.	Newcomb's Precession.
$20''$	$277\frac{1}{2}°$	$278\frac{1}{2}°$	270°	271°	275°	276°
$10''$–$20''$	274	277	266	271	270	274
5–10	$265\frac{1}{2}$	272	262	$272\frac{1}{2}$	264	272
0–5	$264\frac{1}{2}$	$281\frac{1}{2}$	238	286	256	282
0–5	267	292	244	284	259	288

For the stars $0''$–$5''$ two results are given : in the first case equal weight was given to each star, in the second equal areas of the sky had equal weight.

The above table shows that the centennial value of the precession constant n for 1850 lies between

	Struve-Peters' value	$2005''\cdot64$
and	Newcomb's value...	$2005''\cdot11$

A correction of about one-third of the difference to Newcomb's value will make a very good agreement between the stars of large

and small proper motions both for the right ascensions and declinations.

In the last section a correction of $+0''\cdot70$ was found to Newcomb's centennial value of m ($4607''\cdot11$). This implies a correction to n of $\frac{20}{46} \times 0''\cdot70$ or $+0''\cdot30$. The present reasoning requires a correction of about $\frac{1}{3} \times 0''\cdot53$ or $+0''\cdot17$. We have adopted the correction $+0''\cdot20$, giving for the centennial value of the precessional constants m and n for 1850.

$$\left.\begin{aligned} m &= 4607''\cdot57 \\ n &= 2005\cdot31 \end{aligned}\right\}$$

which amount to increasing Newcomb's value by $\frac{1}{10000}$th part.

The following values are derived from the discussion of the proper motions corrected for this new value of the precession and the systematic correction $\Delta\alpha_\delta$, as explained above

Centennial P.M.	From R.A.	From Dec.	Combined Result.
$<20''$	$278°$	$272°$	$275\frac{1}{2}°$
$10''$–$20''$	276	269	$273\frac{1}{2}$
5–10	$269\frac{1}{2}$	$268\frac{1}{2}$	269
0– 5	$\left\{\begin{matrix}277 \\ 283\frac{1}{2}\end{matrix}\right.$	$\left\{\begin{matrix}265\frac{1}{2} \\ 268\frac{1}{2}\end{matrix}\right.$	$\left\{\begin{matrix}273 \\ 278\end{matrix}\right.$
0– 5			

For comparison with the new value of n for 1850 the following values are taken from Newcomb's Precessional Constant, p. 10 :—

Bessel I.	$20''\cdot0413$	Newcomb (prelim. value)	$20''\cdot0479$
Bessel II.	20·0553	Dreyer	20·0546
Peters	20·0564	Bolte	20·0537
Leverrier	20·0524	Newcomb	20·0511
Oppolzer	20·0515	Dyson & Thackeray	20·0531
L. Struve	20·0452		

V. *The Solar Motion.*

The difficulties which are inherent in the determination of the direction of the solar motion are illustrated to some extent in the statistics of Section II. We found there—

(i.) Want of uniformity in the distribution of the stars.
(ii.) Want of uniformity in the distribution of the proper motions.

To elucidate the extent of the uncertainty of the result which might be expected from these indications of systematic differences

in the distances of the stars in different parts of the sky, separate determinations are made in this section for—

(i.) Stars between definite limits of proper motion.
(ii.) Stars of different magnitudes.
(iii.) Stars of Types I. and II.

As regards the formulæ to be used, the simple and convenient method introduced by Airy has been employed. If X, Y, Z are the coordinates of the solar apex, then each proper motion in right ascension leads to an equation of the form $X \sin \alpha - Y \cos \alpha = \mu$; and each proper motion in north polar distance to an equation of the form

$$X \cos \alpha \sin \delta + Y \sin \alpha \sin \delta - Z \cos \delta = -\mu'.$$

The data on which the solutions are based are as follows :— Nine stars of extremely large proper motion have been entirely omitted.

All the stars whose proper motions are greater than 20″ per century have been treated separately, equations of condition being obtained for each star, and normal equations formed and solved in the usual way.

For stars whose proper motions are less than 20″ a century means of the proper motions in right ascension and north polar distance have been formed for all the stars in areas approximately 10° square according to the following scheme :—

Limits of N.P.D.	Extent in R.A.	Number of Areas.	Limits of N.P.D.	Extent in R.A.	Number of Areas.
° °	h		° °	h	
5–15	2	12	35–45	¾	32
15–25	1½	16	45–52	½	48
25–35	1	24			

The thirty-four stars within 5° of the pole have been omitted When two stars appear to have a common proper motion they have been treated as one star.

There will be found at the end of this paper a list of the nine stars with proper motions too large to be used, a list of the stars with proper motions greater that 20″ a century, also the mean proper motions in right ascension and declination, omitting the stars of large proper motion, for each of the 132 areas into which the sky from the pole to N.P.D. 52° has been divided. The data for the further subdivisions (*e.g.* Types I. and II. stars, &c.) are not given for each group, but mean proper motions for every 3^h of R.A. for the separate zones are given for stars of Types I. and II. spectra. All these proper motions here given are determined with Struve-Peters precessions.

The following solutions, however, depend on proper motions deduced with the new value of the precession (p. 443) and cor-

rected further by the systematic corrections in R.A. given on p. 440. All the quantities are given in terms of centennial proper motions.

The general character of the normal equations is shown by the following example found for the 4001 stars whose proper motions are $< 20''$, giving equal weight to each star :—

From Right Ascensions.

$$\left.\begin{aligned} 195{\cdot}8X + 24{\cdot}6Y &= -\ 6''{\cdot}3 \\ 24{\cdot}6X + 205{\cdot}2Y &= -439{\cdot}0 \end{aligned}\right\}$$

From Declinations.

$$\left.\begin{aligned} 133{\cdot}8X - 12{\cdot}9Y - 43{\cdot}2Z &= -\ 18''{\cdot}4 \\ -12{\cdot}9X + 122{\cdot}6Y + 16{\cdot}9Z &= -202{\cdot}9 \\ -43{\cdot}2X + 16{\cdot}9Y + 144{\cdot}4Z &= +168{\cdot}1 \end{aligned}\right\}$$

From Right Ascensions and Declinations.

$$\left.\begin{aligned} +329{\cdot}6X + 11{\cdot}7Y - 43{\cdot}2Z &= -\ 24''{\cdot}7 \\ +\ 11{\cdot}7X + 327{\cdot}7Y + 16{\cdot}9Z &= -641{\cdot}9 \\ -\ 43{\cdot}2X + 16{\cdot}9Y + 144{\cdot}4Z &= +168{\cdot}1 \end{aligned}\right\}$$

General Solution. Proper Motions $0''$–$20''$. 4001 *Stars.*

(Equal weights to each star.)

	X	Y	Z	M.	A.	D.
	$''$	$''$	$''$	$''$	$^\circ$	$^\circ$
From R.A. ...	+0·33	− 2·00	...	...	276	...
„ Decl. ...	+0·13	− 1·85	+ 1·43	...	274	38
R.A. and Decl.	+0·19	− 1·90	+ 1·47	2·40	275	38

Proper Motions $0''$–$20''$. 4001 *Stars.*

(Equal weight to equal areas.)

	$''$	$''$	$''$	$''$	$^\circ$	$^\circ$
From R.A. ...	+0·13	− 2·30	...	...	273	...
„ Decl. ...	+0·01	− 1·90	+ 1·27	...	270	34
R.A. and Decl.	+0·09	− 2·09	+ 1·27	2·55	272	31

(i.) *Solutions according to Size of Proper Motion.*

(*a*). *Proper Motion* $0''$–$5''$. 2885 *Stars.*

(Equal weight to each star.)

	X	Y	Z	M.	A.	D.
	$''$	$''$	$''$	$''$	$^\circ$	$^\circ$
From R.A. ...	+0·13	− 1·04	...	...	277	...
„ Decl. ...	−0·07	− 0·70	+ 0·68	...	$264\frac{1}{2}$	44
R.A. and Decl.	+0·05	− 0·91	+ 0·74	1·17	273	39

(Equal weights to equal areas of the sky.)

	X.	Y.	Z.	M	A.	D.
	″	″	″	″	°	°
From R.A. ...	+0·22	− 0·92	...	...	283½	...
„ Decl. ...	−0·02	− 0·81	+ 0·58	...	268½	35
R.A. and Decl.	+0·12	− 0·86	+ 0·58	1·08	278	34

(*b*). *Proper Motion* 5″–10″. 800 *Stars.*

From R.A. ...	−0·04	− 3·68	...	...	269½	...
„ Decl. ...	−0·10	− 3·46	+ 2·61	...	268½	38½
R.A. and Decl.	−0·05	− 3·56	+ 2·69	4·47	269	37½

(*c*). *Proper Motion* 10″–20″. 316 *Stars.*

From R.A. ...	+0·81	− 7·63	...	...	276	...
„ Decl. ...	−0·14	− 8·54	+ 3·33	...	269	21
R.A. and Decl.	+0·45	− 7·92	+ 3·20	8·36	273½	22

(*d*). *Proper Motion* >20″. 163 *Stars.*

From R.A. ...	+3·10	−21·83	...	...	278	...
„ Decl. ...	+0·67	−20·88	+12·62	...	272	31
R.A. and Decl.	+2·12	−21·51	+12·74	25·11	275½	30½

The stars from 10″–20″ give a very small value for D. When the above results are combined it is found that

D = 27° from stars of p.m. >10″

and D = 39° „ „ „ <10″

(ii.) *Solutions for Stars of Different Magnitudes.*

A determination was made for stars brighter than $6^{m}\cdot5$, excluding those whose proper motion was greater than 20″ a century, and gave the following results :—

Stars of $6^{m}\cdot5$ *and brighter, p.m.* 0″–20″. 1382 *Stars.*

	X.	Y.	Z.	M.	A.	D.
	″	″	″	″	°	°
From R.A. ...	−0·05	−2·64	...	...	269	...
„ Decl. ..	+0·11	−2·14	+1·50	...	273	35
R.A. and Decl.	0·00	−2·47	+1·51	2·90	270	32

Rigorous solutions have not been made for the separate magnitudes, but the following rough determinations from the tables on p. 437 are of interest. Equal weight has been given to each octant, and the sine and cosine of the mean declination have been taken as 0·8 and 0·6. Stars of proper motion >20″ a century have been omitted.

Mags. 1·0^m–4^m·9. 200 *Stars.*

	X.	Y.	Z.	M.	A.	D.
	"	"	"	"	°	°
From R.A. ...	−1·67	−2·72	...	...	232	...
„ Decl. ...	−0·94	−3·76	+0·87	...	266	13
R.A. and Decl.	−1·46	−3·02	+0·87	3·96	245	16

Mags. 5^m·0–5^m·9. 454 *Stars.*

	"	"	"	"	°	°
From R.A. ...	−0·01	−2·83	...	...	270	...
„ Decl. ...	−0·22	−2·86	+1·43	...	265½	26½
R.A. and Decl.	−0·09	−2·84	+1·43	3·18	268	27

Mags. 6^m·0–6^m·9. 1003 *Stars.*

	"	"	"	"	°	°
From R.A. ...	+0·52	−2·53	...	...	281½	...
„ Decl. ...	+0·08	−2·02	+1·55	...	272	37½
R.A. and Decl.	+0·36	−2·39	+1·55	2·85	278	33

Mags. 7^m·0–7^m·9. 1239 *Stars.*

	"	"	"	"	°	°
From R.A. ...	+0·55	−1·88	...	...	286	...
„ Decl. ...	−0·07	−1·50	+1·40	...	267½	43
R.A. and Decl.	+0·32	−1·74	+1·40	2·24	280	38½

Mags. 8^m·0–8^m·9. 811 *Stars.*

	"	"	"	"	°	°
From R.A. ...	−·06	−1·81	...	...	268	...
„ Decl. ...	+·27	−1·54	+1·57	...	280	46½
R.A. and Decl.	+·06	−1·70	+1·57	2·32	272	43

The most remarkable feature of the above table is the progressive increase of D as fainter stars are taken. The agreement of the results from right ascensions and declinations is throughout satisfactory, so that there is no reason to suppose the result is due to any systematic errors of the observations.

The results may be compared with those from Bradley's stars given by Newcomb, "The Precessional Constant" (pp. 31 and 32).

(iii.) *Solutions for Stars of Spectral Types I. and II.*

The stars of Type I. are obtained from the Draper Catalogue as explained on p. 434.

Stars of Type I.

p.m. < 20″. Mean Mag. 6·3. 1100 Stars.

	X.	Y.	Z.	M.	A.	D.
	"	"	"	"	°	°
From R.A.	+0·12	−2·48	...	...	273	...
„ Decl.	−0·34	−1·85	+0·88	...	263	25
Combined result	−0·07	−2 25	+0·95	2·44	269	23

The table on p. 434 shows how uniformly the stars of Type II. are distributed over the part of the sky with which we are concerned. The proportion of stars between different limits of proper motion is also very regular. On this account these stars would seem to be very suitable for a determination of the direction of the Sun's motion, as the natural inference from their symmetrical distribution both in numbers and proper motions is that they are actually in space distributed symmetrically with reference to the Sun.

Stars of Type II.

p.m. < 20″. Mean Mag. $6^{m}\cdot2$. 866 Stars.

	X.	Y.	Z.	M.	A.	D.
	″	″	″	″	°	°
From R.A.	+0·05	−2·28	...	...	271	...
„ Decl.	+0·25	−2·20	+1·65	...	276	37
Combined result	+0·13	−2·23	+1·66	2·79	273	37

p.m. 20″–40″. 89 Stars.

	″	″	″	″	°	°
From R.A.	+1·0	−19·3	...	...	273	...
„ Decl.	+0·9	−20·0	+13 1	...	272½	33
Combined result	+0·9	−19·6	+13·0	23·5	272½	34

Remainder of Stars, omitting those used in Solutions for Types I. and II.

p.m. <20″. 2035 Stars.

	X.	Y.	Z.	M.	A.	D.
	″	″	″	″	°	°
From R.A. ...	+0·35	−1·97	...	...	280	...
„ Decl. ...	+0·35	−1·70	+1·56	...	282	42
Combined result	+0·33	−1·81	+1·58	2·42	280	41

This solution is formed by taking the stars used for Types I. and II. out of the general solution of 4001 stars p.m. 0″–20″ on p. 445.

As referring generally to the different solutions it has been found that another discordance to which attention should be drawn is that the value of A, the right ascension of the solar apex, is smaller when found from the stars between 6^{h}–18^{h} than those between 18^{h}–6^{h}. The values found are—

6^{h}–18^{h}.	18^{h}–6^{h}.	
°	°	
262	288	Stars of p.m. > 20″.
250	284	Stars of Type II. of p.m. > 20″.
270	277	Stars of p.m. 10″–20″.
264	276	Stars of p.m. 5″–10″.

The following seem to be the noticeable features in the above

values of the position of the apex of the Sun's way from the different groupings of the stars :

1. The systematic difference in the results given by stars of Type I. and Type II.
2. Faint stars give large values of A and D, while the brightest stars give small values.
3. The progressive values of D under the groupings arranged according to magnitude.
4. Large as compared with small proper motions give a small value of D.
5. The small value of D given by the group of stars limited to 10″-20″.

The different values of A and D are grouped in a table at the beginning of this paper (p. 429), and the value of the position of the apex of the Sun's motion assumed as most probable is

$$A = 275^\circ \qquad D = 37^\circ$$

Before leaving this subject it seems well once again to call attention to the effect of precession and systematic corrections on the values of the coordinates of the position of the apex of the Sun's motion. The relative effects will of course depend on the size of the proper motions under discussion ; but taking for this purpose the general solution of 4001 stars with centennial proper motions ranging between 0″ to 20″ the following results are due to

1. *Precession.*—The effect on D is small and uncertain, but on A considerable, thus :

Precession	A
Struve-Peters	266°
Adopted value	275
Newcomb	279

2. The systematic corrections in R.A. as given on p. 440.

diminish A between 2° and 3°
also D by 1°

3. A constant systematic error in the N.P.D.s would alter the value of Z by twice the error ; thus a correction of 0″·1 would alter Z by 0″·2 and the value of D by 4°.

The following lists represent annual proper motions in arc, in R.A., and N.P.D. deduced directly from a comparison between the Groombridge and Greenwich Catalogues (the latter corrected to Newcomb's system for the purpose) with the Struve-Peters constants of precession.

List of Stars omitted on account of extremely great Proper Motion.

Name.	Magnitude and Spectral Type.		R.A.	N.P.D.	Annual Proper Motion	
					R.A.	N.P.D.
	m		h m	°	″	″
Groomb. 34 ...	8·3		0 10	46·8	+2·850	−0·378
η Cass. ...	3·7	II.	0 40	32·5	+1·100	+0·511
μ Cass. ...	5·3	II.	0 58	35·8	+3·375	+1·551
ι Pers.	4·2	II.	2 58	41·0	+1·243	+0·064
θ Urs. Mag. ...	3·3	II.	9 23	37·6	−0·934	+0·541
Groomb. 1618...	6·8	II.	10 2	39·8	−1·337	+0·515
Groomb. 1830...	6·5	I.	11 44	51·2	+3·985	+5·766
σ Drac. ...	4·8	II.	19 33	20·6	+0·558	+1·762
η Ceph. ...	3·6	II.	20 42	28·8	+0·109	−0·810

List of Stars whose Annual Proper Motion is greater than 0″·200.

β Cassiop. ...	2·4	II.	0 1	31·7	+·533	+·190
23 Androm. ...	5·6	II.	0 6	49·8	−·127	+·163
Groomb. 93 ...	7·4	I.	0 26	42·9	+·393	−·053
Groomb. 126 ...	7·9	II.	0 35	14·9	+·391	+·113
Groomb. 145 ...	7·6		0 40	20·4	+·500	−·217
θ Cass. ...	4·5	I.	1 2	35·7	+·234	+·037
Groomb. 295 ...	7·5		1 12	38·8	+·295	+·108
δ Cass. ...	2·8	I.	1 16	30·6	+·304	+·048
Gr. 307 ...	7·3	II.	1 17	16·6	+·202	+·128
ω Androm. ...	5·2	II.	1 19	45·4	+·327	+·099
Groomb. 356 ...	6·5	II.	1 30	44·9	+·221	+·234
Pi. I. 142 ...	5·0	II.	1 33	48·1	+·798	+·138
Pi. I. 159 ...	5·6	II.	1 37	27·0	+·583	+·227
54 Cass. ...	6·7	II.	1 56	19·2	+·301	+·240
6 Persei ...	5·4	II.	2 4	39·7	+·317	+·153
θ Persei ...	4·1	II.	2 34	41·4	+·346	+·097
κ Persei ...	4·0	II.	2 59	45·7	+·183	+·154
Pi. III. 28 ...	6·2	II.	3 11	41·5	+·212	+·064
Groomb. 706 ...	7·4	I.	3 25	47·6	+·148	+·152
Groomb. 717 ...	7·0	I.	3 29	48·0	−·178	+·115
Groomb. 745 ...	8·2		3 42	14·3	+·377	+·527
Groomb. 762 ...	7·8	II.	3 49	15·2	+·196	+·299
Groomb. 775 ...	7·7		3 54	20·9	+·099	+·283
Groomb. 807 ...	7·5		4 11	12·7	+·224	+·106
Groomb. 864 ...	7·3		4 31	48·2	+·549	+·409
Groomb. 884 ...	7·1	I.	4 41	44·4	+·387	+·563

Name.	Magnitude and Spectral Type.	R.A.	N.P.D.	Annual Proper Motion. R.A.	Annual Proper Motion. N.P.D.
	m	h m	°	″	″
Capella ...	0·2 II.	5 6	44·2	+·083	+·429
λ Aurig. ...	4·9 II.	5 9	50 0	+·522	+·654
18 Camelop. ...	6·5 II.	5 20	32·9	+·118	+·218
Groomb. 990 ...	7·8	5 26	38·7	−·543	−·113
Groomb. 986 ...	7·3 II.	5 27	15·5	+·185	+·173
Piazzi V. 146 ...	6·4 II.	5 29	36·6	+·007	+·511
Groomb. 1108...	7·4	6 3	33·0	−·204	+·194
Piazzi VI. 75 ...	5·7 II.	6 18	31·7	−·027	+·325
6 Lyncis ...	5·7 II.	6 21	10·3	−·057	+·602
Groomb. 1178...	6·6 II.	6 23	31·8	+·112	+·170
Groomb. 1216...	7·8	6 35	45·6	+·168	+·211
18 Lyncis ...	5·4 II.	7 3	30·1	−·094	+·258
Piazzi VII. 132	6·5 II.	7 31	9·4	−·488	−·051
Groomb. 1437 ...	6·5 II.	8 17	43·8	−·062	+·361
ι Urs. Maj. ...	3·1 I.	8 49	41·4	−·454	+·250
10 Urs. Maj. ...	4·0 II.	8 51	47·6	−·463	+·250
Groomb. 1514...	6·6 II.	8 57	51·1	−·206	+·015
Groomb. 1571...	7·3 II.	9 32	40 5	−·075	+·197
15 Leo. Min. ...	5·1 II.	9 39	43·3	+·193	+·092
υ Urs. Maj. ...	3·8 II.	9 40	30·2	−·306	+·161
Groomb. 1596...	8·0	9 51	33·7	−·192	+·455
Piazzi X. 31 ...	6·7 II.	10 10	45·2	+·079	+·294
Groomb. 1646...	6·5 I.	10 19	40·4	+·092	+·867
Piazzi X. 96 ...	7·6	10 25	40·0	+·263	−·140
Groomb. 1666...	6·9 II.	10 29	29 1	−·033	+·209
38 Leo. Min. ...	5·8 II.	10 31	51·3	−·228	+·045
Piazzi X. 135 ...	5·2 I.	10 35	43·0	−·288	+·087
Piazzi X. 137 ...	8·0	10 35	43·0	−·265	+·071
Groomb. 1697...	6·2 II.	10 43	19 4	−·388	+·079
47 Urs. Maj. ...	5·1 II.	10 51	48·7	−·340	−·054
Gr. 1744 ...	8·4	11 3	46·4	−·147	+·233
Gr. 1745 ...	7·3	11 3	46·4	−·139	+·249
Gr. 1766 ...	7·2 II.	11 12	37·4	−·174	+·102
Piazzi XI. 74 ...	5·9 II.	11 20	27·4	−·117	−·251
Groomb. 1794...	6·9	11 23	45·6	−·280	−·073
Groomb. 1796...	6·4 II.	11 23	41·2	−·242	+·071
Groomb. 1812...	6·7	11 30	44·0	−·594	−·015
Groomb. 1822...	7·7	11 38	41·5	−·568	+·292

Name.	Magnitude and Spectral Type.		R.A.	N.P.D.	Annual Proper Motion. R.A.	N.P.D.
	m		h m	°	″	″
67 Urs. Maj. ...	5·2	I.	11 54	46·1	−·333	−·079
Groomb. 1855...	7·4		12 2	48·9	−·316	+·052
Groomb. 1866...	8·9		12 9	25·5	−·269	−·073
Brad. 1656 ...	6·4	II.	12 12	2·8	+·258	+·012
Groomb. 1876...	8·0	II.	12 14	27·4	−·275	+·256
7 Can. Ven. ...	6·2	II.	12 23	37·6	−·262	−·026
β Can. Ven. ...	4·3	II.	12 27	47·8	−·699	−·300
10 Can. Ven. ...	6·0	II.	12 38	49·9	−·379	−·138
α Can. Ven. ...	2·7	II.	12 49	50·9	−·243	−·050
Groomb. 1947...	7·7	II.	12 53	20·4	−·278	−·262
Piazzi XIII. 96	6·5	II.	13 21	26·0	−·398	−·223
Groomb. 2021...	8·4		13 31	50·1	−·199	+·138
Groomb. 2022...	7·8		13 31	50·1	−·210	+·131
Piazzi XIII. 200	6·5	II.	13 40	33·3	+·098	+·360
Groomb. 2068...	6·5	II.	13 53	27·8	−·019	−·210
λ Bootis ...	4·2	I.	14 11	43·2	−·186	−·166
θ Bootis ...	3·9	II.	14 20	37·4	−·243	+·400
g Bootis ...	5·5	II.	14 23	39·5	−·291	+·048
Groomb. 2116...	6·5	I.	14 24	47·5	+·156	+·208
Groomb. 2131...	6·4	II.	14 30	32·2	+·202	+·226
Groomb. 2141...	7·3		14 35	51·2	−·072	−·201
Groomb. 2152...	6·1	I.	14 43	51·6	−·248	−·118
Groomb. 2154...	5·5		14 45	52·1	−·226	−·100
Piazzi XIV. 235	5·7	II.	14 51	39·7	+·077	+·230
i Bootis ...	4·8	II.	14 59	41·8	−·402	−·026
9 Urs. Min. ...	6·7	II.	15 0	17·6	−·375	−·091
Groomb. 2196...	5·7		15 2	6·9	+·197	+·223
Groomb. 2214...	5·1	II.	15 13	22·1	+·209	+·400
Groomb. 2234...	7·5		15 21	28·9	−·359	−·174
Groomb. 2273...	7·3		15 40	36·5	−·242	−·046
Groomb. 2275...	6·5	II.	15 38	9·1	−·199	−·130
Groomb. 2276...	8·3		15 38	9·1	−·164	−·113
χ Herculis ...	4·7	II.	15 47	47·1	+·422	−·616
Groomb. 2296...	4·9	I.	15 55	34·8	−·190	−·123
θ Draconis ...	4·1	II.	15 59	31·0	−·296	−·338
Groomb. 2305...	6·7		16 0	50·4	−·581	−·057
14 Herc. ...	6·5		16 6	45·8	+·125	+·290
η Urs. Min. ...	5·2	II.	16 22	13·9	−·058	−·251

Name.	Magnitude and Spectral Type.		R.A.	N.P.D.	Annual Proper Motion. R.A.	Annual Proper Motion. N.P.D.
	m		h m	°	″	″
Groomb. 2354...	7·0		16 26	41·7	−·109	+·295
Pi. XVI. 195 ...	6·6	II.	16 37	12·3	−·097	−·277
Groomb. 2389...	6·7		16 49	46·9	+·110	+·324
Groomb. 2391...	6·1	II.	16 50	12·2	+·058	−·212
Groomb. 2393...	7·0		16 53	27·7	−·345	+·052
h^1 Drac. ...	4·9	II.	16 55	24·6	+·250	−·051
26 Drac. ...	5·3	II.	17 33	28·0	+·233	+·509
ω Drac. ...	4·9	II.	17 38	21·2	+·016	−·333
30 Drac. ...	5·1	I.	17 45	39·2	−·059	−·207
ψ Drac. ...	4·8	II.	17 45	17·8	+·010	+·278
ψ^2 Drac. ...	6·2	II.	17 45	17·8	+·031	+·295
35 Drac. ...	5·1	II.	17 56	13·0	+·069	−·230
Groomb. 2527...	6·0	II.	18 7	35·8	+·128	−·250
Groomb. 2538...	6·2	II.	18 12	49·1	−·191	−·072
36 Drac. ...	5·1	II.	18 13	25·7	+·381	−·033
Groomb. 2571...	8·4		18 22	44·0	−·329	−·208
χ Drac. ...	3·7	II.	18 24	17·3	+·518	+·380
α Lyræ ...	0·1	I.	18 32	51·4	+·209	−·286
Groomb. 2624...	8·2		18 33	47·5	+·281	−·069
Groomb. 2630...	8·0	I.	18 34	26·4	−·062	+·252
Groomb. 2686...	7·2		18 46	51·6	+·317	−·027
Groomb. 2699...	5·6	II.	18 48	37·2	−·018	−·268
Groomb. 2789...	5·9	II.	19 8	40·4	−·170	−·620
Groomb. 2809...	6·0	II.	19 13	43·3	−·015	−·290
Groomb. 2867...	8·5		19 28	41·7	+·063	−·333
Piazzi XIX. 191	8·2		19 28	40·2	−·095	−·303
Groomb. 2875...	6·7		19 29	31·7	−·543	+·375
Piazzi XIX. 211	5·7	II.	19 30	39·1	+·015	+·208
θ Cygni ...	4·5	II.	19 32	40·1	−·011	−·240
c Cygni ...	5·9	II.	19 38	39·8	−·141	+·143
Brad. 2513 ...	6·2	II.	19 38	39·8	−·137	+·155
Groomb. 2961...	7·7		19 48	51·6	−·016	−·320
Groomb. 3012...	8·3		19 56	27·5	+·186	−·138
Groomb. 3042...	5·7	I.	20 2	37·3	+·211	−·256
Piazzi XX. 30...	8·8		20 4	26·7	+·194	−·157
Groomb. 3127...	8·2		20 13	40·2	−·230	+·206
Groomb. 3150...	6·0	II.	20 16	23·6	+·495	−·287
Groomb. 3215...	7·0		20 28	48·6	−·155	−·436

Name.	Magnitude and Spectral Type.		R.A.	N.P.D.	Annual Proper Motion. R.A.	Annual Proper Motion. N.P.D.
	m		h m	°	″	″
Groomb. 3249...	6·9		20 34	48·3	−·085	+·216
74 Drac. ...	6·1	II	20 38	9·4	+·096	−·207
Piazzi XX. 332	4·7	II.	20 42	33·0	−·084	+·242
Groomb. 3357...	6·7		20 54	50·4	+·225	−·209
Brad. 2792 ...	5·5	I.	21 20	44·0	+·208	−·040
Groomb. 3477...	7·9		21 23	10·3	+·181	−·096
ξ Cephei ...	4·4	I.	21 59	26·1	+·212	−·065
Groomb. 3689...	8·6		22 1	37·6	−·532	−·341
Brad. 2926 ...	5·5	II.	22 6	33·9	+·249	−·140
ε Cephei ...	4·2	I.	22 10	33·7	+·438	−·046
Groomb. 3794...	8·2		22 24	50·1	−·259	−·041
Groomb. 3843...	6·9		22 32	46·5	+·243	−·056
Groomb. 3848...	6·7	II.	22 33	39·7	+·201	−·047
3 Androm. ...	4·8	II.	22 57	40·8	+·144	−·169
6 Androm. ...	5·8	I.	23 3	47·3	−·198	+·192
Brad. 3084 ...	5·7	II.	23 10	37·6	+·103	+·250
Brad. 3094 ...	6·4		23 13	42·5	+·204	−·021
Groomb. 4047...	7·7		23 15	11·8	+·258	−·095
14 Andr. ...	5·3	II.	23 24	51·6	+·281	+·091
λ Andr. ...	3·9	II.	23 30	44·4	+·147	+·431
Groomb. 4107...	7·1		23 31	41·9	+·212	+·119
Pi. XXIII. 218	6·5	II.	23 45	15·3	+·331	−·045
Pi. XXIII. 242	6·9	II.	23 51	40·4	−·082	−·230

Mean Positions and Proper Motions of Groups of Stars whose Annual Proper Motions are less than 0″·200.

Number of Stars in Group.	Mean R.A. of Group.	Mean N.P.D. of Group.	Mean Proper Motion. R.A.	Mean Proper Motion. N.P.D.
	Zone 5°–15°.			
	h m	°	″	″
23	1 19	12·0	+·011	−·005
39	3 5	11·3	+0·13	+·014
16	4 55	13·0	+·014	+·019
28	7 17	11·3	+·002	+·020
7	8 54	9·2	−·033	+·003
22	11 6	10·5	−·032	+·003
18	12 52	8·4	−·007	−·012
22	1 56	11 6	−·004	−·011
17	16 50	11·7	−·008	−·011
13	18 55	11·0	+·009	−·009
24	21 6	11·0	+·019	+·001
32	22 44	11·6	+·018	−·004
	Zone 15°–25°.			
	h m	°	″	″
56	0 46	20·2	+·025	+·008
55	2 15	19·2	+·021	+·020
44	3 48	19·4	+·020	+·015
36	5 15	19·4	+·016	+·020

Number of Stars in Group.	Mean R.A. of Group.	Mean N.P.D. of Group.	Mean Proper Motion. R.A.	Mean Proper Motion. N.P.D.
	h m	°	″	″
22	7 8	20·7	−·001	+·019
40	8 14	20·8	−·012	+·027
14	9 48	20·7	−·030	+·025
21	11 8	21·3	−·008	+·013
26	12 56	20·2	−·027	+·006
25	14 9	21·4	−·009	−·008
31	15 42	20·4	−·004	·000
24	17 8	18·6	−·003	+·002
15	18 47	21·0	+·009	−·021
33	20 21	20·4	+·013	+·002
50	21 52	20·3	+·024	+·005
59	23 24	19·1	+·021	+·004
	Zone 25°–35°.			
	h m	°	″	″
46	0 39	30·0	+·017	+·010
28	1 33	28·3	+·019	+·010
13	2 21	32·9	+·018	+·002
17	3 33	29·0	−·002	−·003
21	4 27	30·2	+·011	+·029
30	5 40	31·5	+·005	+·031
48	6 29	31·6	−·003	+·020
38	7 35	31·4	−·009	+·026
19	8 36	28·6	−·015	+·019
20	9 35	30·5	−·015	+·006
22	10 36	31·4	−·039	+·016
26	11 30	30·5	−·023	−·004
32	12 30	29·8	−·015	−·010
17	13 30	31·5	−·008	+·005
18	14 35	30·2	−·029	+·007
33	15 28	29·9	−·017	−·007
21	16 32	29·4	−·002	−·012
9	17 19	29·6	+·019	−·036
34	18 40	30·5	+·003	−·017
40	19 36	30·8	+·017	−·009
63	20 30	31·2	+·010	−·006
68	21 38	29·5	+·016	−·008
53	22 40	30·4	+·025	+·003
45	23 31	29·6	+·010	+·005

Number of Stars in Group.	Mean R.A. of Group.	Mean N.P.D. of Group.	Mean Proper Motion. R.A.	Mean Proper Motion. N.P.D.
	Zone 35°–45°.			
	h m	°	″	″
61	0 24	40·1	+·011	+·010
60	1 6	41·7	+·017	+·013
63	1 55	39·2	+·001	+·017
31	2 41	39·5	+·008	+·012
54	3 21	40·9	+·021	+·023
30	4 12	39·4	+·011	+·028
30	4 52	40·0	+·003	+·036
54	5 43	40·5	+·002	+·020
23	6 18	40·7	−·010	+·017
24	7 9	40·5	−·012	+·045
9	7 49	37·7	−·032	+·011
17	8 40	39·7	−·038	+·024
23	9 19	39·9	−·012	+·014
20	10 5	39·4	−·020	+·004
27	10 55	41·0	−·028	+·020
10	11 36	41·0	−·005	+·002
10	12 27	40·1	−·033	−·028
17	13 13	41·0	−·015	+·002
16	13 46	39·2	−·039	−·015
17	14 43	40·5	−·012	+·009
20	15 25	39·5	−·007	·000
22	16 13	39·9	−·016	−·001
7	16 45	40·0	−·023	−·026
23	17 47	42·3	−·007	+·010
48	18 24	40·5	+·004	−·007
67	19 11	40·0	+·004	+·004
63	19 57	40·1	+·005	−·007
90	20 37	39·4	+·011	+·002
65	21 24	39·0	+·008	−·002
92	22 7	38·9	+·009	+·006
71	22 49	39·7	+·019	+·007
64	23 41	39·5	+·017	+·013
	Zone 45°–52°.			
	h m	°	″	″
33	0 16	47·0	+·006	+·014
31	0 47	47·7	+·013	+·021

Number of Stars in Group.	Mean R.A. of Group.	Mean N.P.D. of Group.	Mean Proper Motion.	
			R.A.	N.P.D.
	h m	°	″	″
22	1 17	47·5	+·017	+·022
17	1 48	48·7	+·009	+·024
10	2 12	48·8	−·016	+·018
20	2 46	48·7	+·004	+·031
27	3 18	47·6	+·017	+·017
11	3 40	48·5	+·003	+·013
14	4 20	48·3	+·004	+·026
11	4 50	48·4	+·004	+·016
28	5 19	49·0	−·005	+·023
22	5 49	47·9	+·004	+·034
40	6 19	48·4	−·009	+·020
25	6 47	48·5	−·012	+·031
21	7 16	49·7	−·021	+·024
8	7 50	47·1	−·021	+·012
14	8 21	47·8	−·030	+·029
20	8 50	48·9	−·029	+·035
11	9 18	48·7	−·033	+·018
16	9 53	49·7	−·051	+·012
18	10 17	47·8	−·051	+·018
18	10 47	47·7	−·044	+·015
25	11 17	47·8	−·030	+·004
9	11 47	48·2	−·049	+·025
14	12 17	48·8	−·036	−·005
8	12 50	48·1	−·043	−·002
21	13 15	49·3	−·043	−·007
14	13 45	48·1	−·035	+·014
19	14 16	49·1	−·025	−·003
13	14 42	48·8	−·035	−·007
14	15 19	48·7	−·018	−·029
8	15 45	48·1	−·019	−·048
8	16 15	49·0	−·027	−·009
9	16 46	48·2	−·025	−·001
8	17 15	50·0	−·017	+·011
30	17 47	48·2	−·008	−·011
42	18 18	48·5	−·002	+·001
81	18 46	48·8	−·011	+·003
42	19 16	48·6	+·001	+·004
77	19 46	49·2	−·005	+·007
81	20 15	48·8	+·002	+·005
45	20 47	48·2	+·017	−·003
34	21 20	47·6	+·002	+·008
40	21 44	48·5	+·006	+·008
21	22 18	48·9	−·001	+·008
41	22 46	48·3	+·019	+·003
27	23 15	48·1	+·015	+·013
34	23 48	48·3	+·010	+·014

Comparison of Mean Proper Motions of Stars of Type I. and Type II. Spectrum for each 3ʰ of R.A. in Zones of N.P.D.

Zone 5°–15°.

Limits of R.A.	Type I. Mean Proper Motion.		Type I. No. of Stars.	Type II. Mean Proper Motion.		Type II. No. of Stars.
	R.A.	N.P.D.		R.A.	N.P.D.	
h h	″	″		″	″	
0– 3	+·020	+·012	17	·000	+·009	14
3– 6	+·013	+·020	10	+·009	·000	13
6– 9	−·007	+·016	12	−·006	+·024	12
9–12	−·067	−·006	4	−·038	+·012	14
12–15	·000	−·011	6	−·017	−·015	14
15–18	−·004	−·013	8	+·012	−·015	13
18–21	+·017	+·010	9	+·029	−·019	9
21– 0	+·025	+·006	17	+·008	−·013	22

Zone 15°–25°.

Limits of R.A. h h	Mean Proper Motion. R.A.	N.P.D.	No. of Stars.	Mean Proper Motion. R.A.	N.P.D.	No. of Stars.
	"	"		"	"	
0– 3	+·023	+·012	43	+·021	+·022	21
3– 6	+·018	+·023	25	+·014	+·009	13
6– 9	−·025	+·035	13	·000	+·017	23
9–12	−·019	−·005	11	−·007	+·027	12
12–15	+·005	·000	12	−·042	·000	21
15–18	−·010	−·012	15	−·007	−·003	16
18–21	+·008	−·006	13	+·016	−·004	15
21– 0	+·027	−·003	38	+·024	+·014	22
		Zone 25°–35°.				
0– 3	+·022	+·012	25	·000	+·002	19
3– 6	+·004	+·024	22	+·001	+·020	19
6– 9	−·007	+·012	35	−·018	+·039	30
9–12	−·029	−·002	15	−·026	+·012	35
12–15	−·001	−·005	24	−·021	+·004	19
15–18	+·019	−·014	17	−·024	−·017	15
18–21	+·010	−·015	30	+·023	−·022	24
21– 0	+·017	·000	54	+·012	+·001	26
		Zone 35°–45°.				
0– 3	+·010	+·012	85	+·012	+·024	40
3– 6	+·012	+·028	72	+·016	+·033	28
6– 9	−·029	+·020	19	−·015	+·031	26
9–12	+·004	−·004	20	−·036	+·008	23
12–15	−·035	−·018	20	−·023	−·004	18
15–18	−·006	+·003	26	−·022	+·009	20
18–21	+·012	−·001	54	−·004	+·001	43
21– 0	+·011	+·009	53	+·020	+·005	34
		Zone 45°–52°.				
0– 3	+·023	+·021	41	−·018	+·025	22
3– 6	+·004	+·022	41	+·014	+·031	26
6– 9	−·006	+·022	24	−·025	+·029	37
9–12	−·060	+·020	18	−·061	+·006	19
12–15	−·064	−·021	17	−·039	+·009	22
15–18	−·027	−·025	17	−·008	−·014	31
18–21	+·003	+·003	56	−·002	−·001	27
21– 0	+·018	+·012	55	+·031	+·006	12

The Determination of Selenographic Positions and the Measurement of Lunar Photographs.

[Fourth Paper.]

First Attempt to Determine the Figure of the Moon.

By S. A. Saunder, M.A.

§ 1. *Introduction.*

In the third paper of this series (*Memoirs R.A.S.* vol. lvii. part i.) I have given the places of 1433 points as determined from the measures of four Paris negatives, the reductions being made on the supposition that the points all lie on the surface of a sphere. But one of the results I hope will follow from the measures on which I am engaged is a determination of the true figure of the Moon; and although the work is at present incomplete, and the plates already measured are, when taken by themselves, not very well suited for such a determination, I have wished to see whether the results obtained are such as to justify a hope that this object may be ultimately accomplished.

It was first pointed out by Newton that if the Moon were originally fluid the tide raised by the Earth should have caused the diameter directed towards us to be longer than that at right angles to it. He computed the elongation to be 186 feet. Summaries of various attempts which have been made to determine this elongation by observation are given by Franz in *Die Figur des Mondes*, pp. 2, 3, 33; and by Mainka in *Breslau Mitteil.* vol. i. pp. 55, 56. They may be divided into dynamical methods which do not necessarily give the geometrical elongation, methods depending on the displacement of the terminator whose position it is always difficult to estimate, and methods depending on the apparent change of position of lunar formations under varying libration.

The results obtained by the geometrical methods are very conflicting. The greatest value of the elongation is that of Gussew, who thought that the part of the Moon towards the Earth was spherical, with a radius 0·982 of the radius of the periphery, but with its centre 0·0726 of the same radius nearer to the Earth, so that, measuring from the centre of gravity of the Moon, the radius towards the Earth was 1·05 times that at right angles to it. The least value is that found by Franz in *Die Figur des Mondes*: he assumes a spheroidal figure and finds an elongation ·00114±·00390 towards the Earth.

More recently Professor W. H. Pickering has found ·0013 ±·0012 by a similar method applied to the measures of twenty of the points given by Franz in *Breslau Mitteil.* vol. i. But as the ·0013 is obtained by dividing the mean altitude of twenty craters "within half a radius of the mean centre of the disc"

above "the mean sphere" by the radius of the Moon, it seems to me to represent a smaller quantity than the excess of the greatest radius over the least. If the greatest radius is really that towards the Earth, the mean of these twenty radii must be less than the greatest; whilst no reason is given for supposing that the radius of the mean sphere is less than the least radius of the Moon (*Harvard Annals*, vol. li. p. 38).

More recently still Hayn in *Ast. Nach.* No. 3956, assuming that *Mösting A* is on the mean surface, finds an elongation of 4000 metres, giving an elongation ·0023. But he points out that this assumption cannot be proved.

The value obtained in the present paper is $\cdot00052 \pm \cdot00027$. This determination is made from a consideration of the absolute altitudes of thirty-eight points measured on each of four negatives, and all situated near the central meridian. The probable error is considerably less than that of any previous determination with which I am acquainted; but although I do think that it shows that the elongation is very small, I do not wish to lay any great stress on the actual result itself. The number of points employed is small, and the individual altitudes are subject to considerable uncertainty. My desire is rather to give grounds for my opinion that the method adopted is one of considerable promise.

§ 2. *Theory of the Method of Determining Absolute Altitudes and of the Apparent Change of Position of a Point under Different Librations.*

The theory of the method I purpose to adopt may be stated as follows :—

Let M be the centre of the Moon.

E the point of observation.

S a point whose altitude is to be determined.

Let the radius MS cut a "mean sphere" whose surface nearly coincides with that of the Moon in B.

Let ES cut the same sphere in P.

In the method of reduction which I have adopted, as well as in that adopted by Dr. Franz, it is assumed that all the observed points are on the surface of a sphere. Suppose this to be the sphere whose radius is MB, then the reduced coordinates ξ, η are those of P.

Let M be taken as origin.

ME as axis of z.

x, y, z the coordinates of P.

$x+\delta x$, $y+\delta y$, $z+\delta z$ those of B.

Let MB = 1, ME = d, BS = h.

The coordinates of S are

$$(x+\delta x)(1+h),\ (y+\delta y)(1+h),\ (z+\delta z)(1+h).$$

The coordinates of E are

$$0,\ 0,\ d.$$

Since the three points E, S, P are in a straight line, ∴ their projections on any one of the coordinate planes are in a straight line.

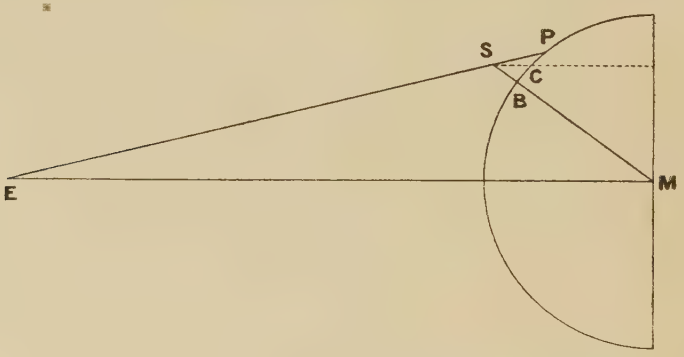


Fig. 1.

Hence

$$\begin{vmatrix} 0, & x, & (x+\delta x)(1+h) \\ d, & z, & (z+\delta z)(1+h) \\ 1, & 1, & 1 \end{vmatrix} = 0\ ; \quad \begin{vmatrix} 0, & x, & (x+\delta x)(1+h) \\ 0, & y, & (y+\delta y)(1+h) \\ 1, & 1, & 1 \end{vmatrix} = 0.$$

These respectively reduce to

$$x\delta z+(d-z)\delta x+\frac{h}{1+h}xd=0$$

$$x\delta y-y\delta x=0$$

And since B, P are both on the sphere whose radius is unity,

$$\therefore\ x^2+y^2+z^2=(x+\delta x)^2+(y+\delta y)^2+(z+\delta z)^2=1.$$

If we put $$h'=\frac{h}{1+h}$$

neglect squares of δx, δy, δz

and remember that $$\frac{1}{d}=\sin s'$$

we get

$$\delta x = -\frac{xzh'}{z-\sin s'}$$

$$\delta y = -\frac{yzh'}{z-\sin s'}$$

$$\delta z = \frac{h'(1-z^2)}{z-\sin s'}$$

If now ξ, η are the coordinates of P referred to the standard axes

$\xi+\delta\xi$, $\eta+\delta\eta$ those of B

we have

$$\xi = Ax+By+Cz, \quad \eta = Dx+Ey+Fz$$

where the direction cosines A, B, C, D, E, F have the values defined in *Monthly Notices R.A.S.* vol. lx. p. 187,

and

$$\delta\xi = A\delta x+B\delta y+C\delta z, \quad \delta\eta = D\delta x+E\delta y+F\delta z.$$

$$\therefore \quad \delta\xi = \frac{C-\xi z}{z-\sin s'}h' = Ph' \text{ say} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\delta\eta = \frac{F-\eta z}{z-\sin s'}h' = Qh' \quad \text{,,} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

If now ξ_1, η_1 are the values of ξ, η given by any particular plate,

P_1, Q_1 the corresponding values of P, Q,

$\bar{\xi}$, $\bar{\eta}$ the mean of the values given by all the plates,

$\bar{\xi}+\delta\bar{\xi}$, $\bar{\eta}+\delta\bar{\eta}$ the coordinates of B,

we have the following conditional equations from which to find $\delta\bar{\xi}$, $\delta\bar{\eta}$, and h :

$$\bar{\xi}+\delta\bar{\xi} = \xi_1+P_1h'$$

$$\bar{\eta}+\delta\bar{\eta} = \eta_1+Q_1h'$$

with similar equations for each of the other plates.

These equations may be written

$$\delta\bar{\xi}-P_1h'+(\bar{\xi}-\xi_1) = 0$$

$$\delta\bar{\eta}-Q_1h'+(\bar{\eta}-\eta_1) = 0$$

The solution by least squares is much facilitated if we notice that by definition of $\bar{\xi}$, $\bar{\eta}$

$$\Sigma(\bar{\xi}-\xi_1) = 0, \ \Sigma(\bar{\eta}-\eta_1) = 0$$

and therefore, if n be the number of plates, the normal equations become

$$n\delta\bar{\xi}-\Sigma P_1h' = 0$$

$$n\delta\bar{\eta}-\Sigma Q_1h' = 0$$

$$-\Sigma P_1 \cdot \delta\bar{\xi}-\Sigma Q_1 \cdot \delta\bar{\eta}+\Sigma(P_1^2+Q_1^2)h'-\Sigma\{P_1(\bar{\xi}-\xi_1)+Q_1(\bar{\eta}-\eta_1)\} = 0$$

The final solution being

$$\delta\bar{\xi} = \frac{1}{n}\Sigma P_1 \,.\, h', \text{ with weight } n \ldots \quad \ldots \quad \ldots \quad (3)$$

$$\delta\bar{\eta} = \frac{1}{n}\Sigma Q_1 \,.\, h', \text{ with weight } n \ldots \quad \ldots \quad \ldots \quad (4)$$

$$h' = \frac{\Sigma\{P_1(\bar{\xi}-\xi_1)+Q_1(\bar{\eta}-\eta_1)\}}{\Sigma(P_1^2+Q_1^2)-\frac{1}{n}\{(\Sigma P_1)^2+(\Sigma Q_1)^2\}} \ldots \quad \ldots \quad (5)$$

the denominator of the last fraction being also the weight of h'

and $$h = \frac{h'}{1-h'}$$

The accuracy at present obtained does certainly not require that the solution should be extended to include terms of the order $(\delta x)^2$, &c.

The coordinates which have been tabulated in the catalogue are the values of $\bar{\xi}$, $\bar{\eta}$; these depend, not only on the position of the point, but also on the librations of the particular plates from which they are determined. It is clear that, when the observations can be made with sufficient accuracy, we should tabulate coordinates which depend only on the position of the point S; but I am not sure that we should endeavour to tabulate those of S itself. If we tabulate those of B then the formulæ for conversion into latitude and longitude will be independent of h, and the same for all points; whilst if we tabulate those of S this will not be true; and, further, if the straight line drawn from S perpendicular to the coordinate plane of ξ, η cut the surface of the Moon again at C, the two points S, C—both on the surface of the Moon—would have the same values of ξ, η, and this might lead to confusion; though both S and C could only become visible to us when B is near the mean centre of the disc. For this reason I think that in mapping it would be desirable to aim at projecting B rather than S.

The coordinates of S are easily obtained by multiplying those of B by $1+h$. It might sometimes be convenient to have them tabulated as well as those of B.

I do not think that we yet know the values of h with sufficient accuracy to really distinguish between the coordinates of B and S; but the discussion has a practical bearing for me, as the conclusion arrived at determines the form of the coefficients P, Q, which I tabulate in my own index catalogue for future use. I should be very glad to receive suggestions on this point from those interested in the matter.

The method which has been adopted in these measures for setting on a crater is such that the recorded position of the centre depends upon the observed position of the crest of its wall.

The radius of the mean sphere adopted for any particular plate is determined in the least square solution for finding the

constants of the plate described in my third paper (*Memoirs R.A.S.* vol. lvii. p. 5). It is therefore a function of the radii drawn to the crests of the walls of all the craters taken as standard points. The altitude found for any particular crater will be that of its crest above or below this mean sphere. I shall eventually assume that a smoothed surface drawn through all these crests will give us an approximation to the figure of the Moon.

§ 3. *Results obtained in the Determination of Absolute Altitudes.*

This preliminary discussion will be confined to the thirty-eight points which have been measured on all four plates, and which are therefore necessarily in the neighbourhood of the Moon's principal meridian. Fourteen of these points have also been measured by Dr. Franz on each of five plates, and I have treated his measures of these fourteen points, as given in *Breslau Mitteil.* vol. i., in precisely the same manner as my own.

The results obtained are exhibited in the following table, where

The first column gives the reference number to the formation as recorded in the complete catalogue.

The second column gives the name of the formation.

The third and fourth columns give its approximate position in selenographical coordinates.

The fifth column gives the absolute altitude found from the measures, with its probable error. These are expressed in terms of a unit equal to Moon's radius $\times 10^{-5}$, which is about 57 feet.

The sixth column gives the corresponding quantities as deduced from Dr. Franz's measures.

The seventh column requires some explanation. The mean altitude of the fourteen points, which both Dr. Franz and I have measured, is $+5$ units according to my measures, and $+175$ units according to Dr. Franz's. This may be taken to mean that the two altitudes are not referred to the same mean sphere, the radius of Dr. Franz's being 170 units less than mine. I have therefore subtracted 170 units from Dr. Franz's altitudes in order to refer them to the same sphere as mine, and the seventh column contains the excess of my altitude over his when so referred. A more complete discussion of the questions involved will be given in the next section.

The eighth and ninth columns contain the probable errors of ξ and η, as determined from the residuals before the correction for altitude is applied. ξ, η here denote the quantities represented by $\bar{\xi}$, $\bar{\eta}$ in § 2, or the coordinates of the mean position of P in fig. 1.

The tenth and eleventh columns contain the corresponding probable errors after the correction for altitude has been applied. ξ, η here denote the quantities represented by $\bar{\xi}+\delta\bar{\xi}$, $\bar{\eta}+\partial\bar{\eta}$ in § 2, or the coordinates of the point B in fig. 1.

1	2	3	4	5	6	7	8	9	10
							Probable Errors of C ordinates.		
Cat. No.	Name.	Approximate Position.		Absolute Altitude and Probable Error.		Difference of Computed Altitudes.	Before Correction for Altitude.		After Co tion f Altitu
		ξ	η	S.A.S.	J.F.	S−(F−170).	ξ	η	ξ
646	Deluc *d*	−·023	−·832	− 89± 21	...	...	± 8·3	± 4·7	± 2·2 ±
648	Deluc H	−·021	−·810	− 63± 39	...	...	8·7	6·4	5·7
712	[near Lilius]	+·023	−·804	− 57± 36	...	...	4·9	8·3	2·8
738	Licetus H	+·038	−·718	−163± 51	...	...	12·2	7·4	4·6
571	Orontius *d*	−·083	−·635	−110± 28	...	...	3·8	5·9	1·8
772	Stöfler K	+·057	−·635	− 57± 113	...	...	7·8	18·7	7·1
605	Orontius *c*	−·056	−·615	− 49± 47	...	...	4·1	7·6	3·0
592	Purbach B	−·066	−·453	−120± 69	...	...	10·8	4·3	9·6
638	Purbach A	−·030	−·440	−311± 50	...	...	13·7	7·7	6·3
729	Lacaille D	+·035	−·401	−283± 65	...	...	13·6	8·1	5·4
576	Thebit A	−·079	−·368	− 11± 26	+115± 59	+ 44	1·0	3·6	0·9
643	Arzachael A	−·024	−·309	−265± 76	...	...	9·8	11·6	1·7
513	Alpetragius B	−·115	−·261	−115± 48	...	...	4·9	6·5	4·2
655	Ptolemæus A	−·014	−·148	−148± 65	+ 55± 55	− 33	7·1	7·6	4·8
770	Hipparchus I	+·056	−·132	+ 27± 76	...	...	4·7	8·9	5·1
736	Hipparchus K	+·038	−·121	+ 60± 37	...	...	3·8	3·7	2·5
507	Lalande D	−·119	−·097	−127± 60	+171± 51	−128	7·7	5·6	6·5
888	Hipparchus G	+·129	−·088	− 18± 65	+279± 30	−127	3·4	6·9	3·6
607	Herschel *c*	−·055	−·087	+ 54± 47	+258± 45	− 34	5·7	3·4	5·0
747	Hipparchus F	+·043	−·073	− 35± 63	...	...	3·5	7·6	3·4
555	Mösting A	−·090	−·056	− 56± 67	+105± 39	+ 9	6·0	6·4	5·7
763	Hipparchus H	+·053	−·039	−108± 49	...	...	5·2	3·6	4·4
684	Triesnecker B	+·007	+·020	+ 98± 58	+193± 69	+ 75	6·5	5·3	5·4
823	Rhæticus A	+·091	+·030	+ 24± 80	+222± 60	− 28	7·3	7·0	7·2
703	Murchison A	+·020	+·070	+ 53± 47	+160± 89	+ 63	3·6	5·3	3·3
779	Triesnecker	+·063	+·073	+ 28± 32	+247± 37	− 49	3·0	3·0	2·2
622	Bode	−·042	+·117	+160± 68	+136± 70	+194	8·7	6·1	7·1
610	Bode B	−·053	+·152	+ 59± 56	+148± 51	+ 81	4·0	6·3	3·0
650	Bode A	−·020	+·156	+ 25± 45	+172± 64	+ 23	5·0	3·1	4·8
542	Archimedes A	−·098	+·470	− 70± 27	+189± 65	− 89	3·1	3·0	1·4
647	Archimedes *c*	−·022	+·524	− 98± 59	...	...	8·4	4·7	6·0
639	Archimedes *b*	−·027	+·570	− 83± 43	...	...	6·2	4·5	4·3
580	Kirch	−·076	+·632	− 42± 23	...	...	3·3	3·3	2·4
756	[near Cassini]	+·049	+·660	−121± 37	...	...	6·8	3·7	5·1
623	Piazzi Smyth	−·042	+·667	− 44± 37	...	...	4·0	4·7	2·6
649	Plato H	−·020	+·820	− 54± 36	...	...	6·0	5·2	4·9
631	Anaxagoras a	−·037	+·952	+ 24± 29	...	...	9·7	4·6	9·4
612	Anaxagoras	−·050	+·959	+ 32± 54	...	...	19·1	7·8	19·3
	Means	...	...	± 51	...	...	± 6·7	± 6·1	± 4·9
	Mean for 14 points measured by Dr. Franz			± 53	± 56	...	...	...	...

§ 4. *Discussion of Results.*

The mean of the probable errors of the altitudes, whether determined from Dr. Franz's measures or my own, is a little over half a mile. As many of the actual errors may be expected to exceed this, the individual altitudes must be taken as subject to considerable uncertainty. The same conclusion may be drawn from an inspection of the seventh column.

In order to compare the altitudes deduced from Dr. Franz's measures with those deduced from my own it is necessary to know, first, the level of that part of the crater to which each set of measures applies, and, secondly, the radius of the mean sphere to which each set is referred. Dr. Franz, discussing the first point with regard to his own measures (*Die Figur des Mondes*, p. 26), comes to the conclusion that the observed point lies on the average on the same level as the surrounding country. I have already stated that in my measures the observed point must be taken as lying on the same level as the crest of the crater. On this account, therefore, my altitudes should on the average exceed Dr. Franz's by the average height of the crest of a crater above the surrounding country.

With regard to the second point I believe that the radius of the sphere to which Dr. Franz refers his points is that given by the equation $\sin s = \sin \pi \times \cdot 272410$ (*Die Figur des Mondes*, p. 9, and *Breslau Mitteil.* vol. i. p. 5). So far as I am able to determine the radius of my mean sphere from data given in this paper it exceeds 1·00079 of that given by the equation $\sin s = \sin \pi \times \cdot 272536$ by the average height of the crest of a crater above the mean surface (see § 7).

If we subtract this average height from the radius of my mean sphere we may then suppose that my measured altitudes are those of points on the same level as those measured by Dr. Franz, and, according to the figures just given, the radius of my sphere would be 1·00125 of that adopted by Dr. Franz. This would make the radius of Dr. Franz's sphere 125 of the units adopted in the table less than mine, which agrees with the systematic difference of 170 units as nearly as could be expected when the great uncertainty of the determinations is considered.

The individual differences given in the seventh column would be to some extent affected by the actual heights of the walls of the individual craters, but it does not seem worth while to discuss this any further. The mean value of the differences, considered without regard to sign, is ± 70 units.

It is unfortunate for the purposes of altitude determination that four plates now measured have all positive libration in latitude, though the librations in longitude are well separated. The result of this is that the conditional equations in η, those derived from equation (2) in § 2, have very little effect upon the solution which depends almost entirely upon those in ξ derived from equation (1). This, however, will correct itself in subsequent

work, as the plates I propose to measure next have all negative libration in latitude.

Dr. Franz's five plates exhibit a greater variety of librations, and are in this respect better adapted than my four for determining altitudes. This is shown by a comparison of the weights of the respective determinations. Taking the weight of a coordinate as determined from a single plate as the unit, the weight of a complete altitude determination from my four plates falls as low as 0·026 for *Hipparchus H*, which is very unfavourably situated, but the determinations for a number of points near the centre of the disc had weights 0·039 and 0·040, whilst for Dr. Franz's plates the lowest value found was 0·075.

The 304 residuals on which my determinations of these thirty-eight altitudes depend are those given in the catalogue, except that for this purpose the telescopic measures are omitted and fresh means and residuals have therefore been found for the five points affected by this omission. An examination of these residuals shows that only four of them exceed 0″·5, and that the mean value of their absolute magnitudes, considered without regard to sign, is ±0″·13.

We are therefore dealing with quantities of the same order as those on which the known stellar parallaxes depend, and we have something very different from stellar points to measure. It is perhaps scarcely to be wondered at when both the quantities to be measured, and the weights of the determinations, are so small that there should be considerable difficulty in separating the real displacements from errors of observation. That it has been possible to do so at all is due to the advantage I have had in measuring photographs taken with a telescope of such great focal length as that of the Equatorial Coudé, 18·05 m., which is at the same time of sufficient aperture to allow of short exposures. If the focal length is unduly increased in comparison with the aperture the time of exposure has to be lengthened, and definition is lost. This, it seems to me, is the real reason for the comparative failure of the admirably conceived system of photographs in vol. li. of the *Harvard Annals*.

In order to get some further indication of the value to be attributed to these results the probable errors in the last four columns of the table were computed. The correction for altitude reduces the average probable error of the ξ coordinates from ±6·7 to ±4·9, which is, I think, more than would be accounted for by the introduction of an entirely arbitrary unknown into the equations, and indicates that a real effect of altitude on this coordinate may be detected. The average probable error of η is scarcely affected. This is due to the distribution of librations already noted in consequence of which the coefficients of the equations are such that they have very little effect upon the determinations of altitude, and conversely a change of assumed altitude has very little effect upon the residuals.

I have made no direct comparison of these altitudes with

those given for some of the same craters in *Die Figur des Mondes*; these last depend upon a smaller number of measures made upon the same plates as were used for the measures given by Dr. Franz in *Breslau Mitteil.* vol. i., and for several reasons seem to me to be less trustworthy than these later measures.

§ 5. *Consideration of the Points whose Apparent Positions are most affected by Change of Libration.*

Professor W. H. Pickering has devoted Chapter IV. in *Harvard Annals*, vol. li., to a consideration of the absolute altitudes of twenty points as deduced from Dr. Franz's measures, and bases his selection of these points on the dictum that "the proposed method of determining altitudes can be applied to most advantage to points situated near the centre of the disc." He gives no reason for this statement, which appears to me to be open to considerable doubt.

There can be no question of the geometrical fact that the displacement which the apparent selenographic position of a point can undergo for a given change of libration increases from the centre to the limb.

Thus let M be the centre of the Moon,

AC a mountain at the mean centre of the disc,

BD a mountain of equal height at B.

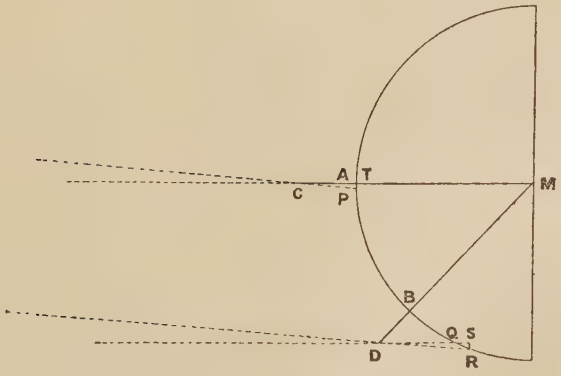


Fig. 2.

If for simplicity we suppose the Earth to be at an infinite distance, then under mean libration the point C will be referred to A on the mean sphere, and D to Q, where DQ is parallel to CA.

If now the Moon be librated through an angle ACP in the plane MAB, so as to obtain the maximum in displacement of Q, C will be referred to P, and D to R where the angle QDR = the angle ACP.

Draw PT perpendicular to MA
and RS „ DQ

Then PT, RS will be the computed displacements of C D respectively, and

$$RS : PT :: DR : CP$$
$$:: \sec BDR : \sec ACP \text{ nearly.}$$

The difference between RS and PT will be increased if we correct for the finite distance of the Earth.

That the effect upon points near the limb is greater than on those near the centre is shown in the weights of the altitudes which rise from 0·026 for *Hipparchus H* to 0·342 for *Anaxagoras*. The general expression for these weights is given in § 2, equation (5) : it depends only on the librations and the position of the point measured. The fact that the probable errors of the determinations, as shown in the fifth column of the table, do not decrease proportionally to the square roots of these numbers is chiefly due to the greater difficulty of measuring a foreshortened crater near the limb where the shadows are much more puzzling ; in part also, perhaps, to some shift of the apparent centre as a formation is brought nearer the limb by change of libration, this shift being due to the sensibly different inclinations of the tangent planes at the nearest and most remote parts of the crater to the plane of projection. But the mean of my probable errors for points within Professor Pickering's selected radius of 0·5 from the centre of the disc is ± 55, whilst for points outside this radius it is ± 44 ; and this seems to me to indicate that the method is more advantageously applied to points remote from the centre of the disc.

§ 6. *Determination of the Figure of the Moon.*

The thirty-eight points whose altitudes have been determined all lie near the central meridian, and can therefore only give us information with regard to the shape of that meridian. We may therefore without loss of generality suppose the figure of the Moon to be represented by a spheroid with its axis of figure directed towards the Earth rather than an ellipsoid with three unequal axes.

The radius of the mean sphere to which the altitudes are referred is not known in terms of the axes of this spheroid. We will take its radius as unity and suppose the semi-axis of the Moon directed towards the Earth to be $1-q+p$, that at right angles to it being $1-q$, where p, q are known to be small.

The equation to the mean surface of the Moon is thus

$$\frac{x^2+y^2}{(1-q)^2}+\frac{z^2}{(1-q+p)^2}=1$$

Neglecting squares of p and q this reduces to

$$(x^2+y^2+z^2)(1+2q)-2pz^2=1$$

Now referring to fig. 1 :

If x, y, z are the coordinates of S on the mean surface

ξ, η, ζ are the coordinates of the corresponding point B on the mean sphere

$BS = h$

Then

$$x=\xi(1+h), \qquad y=\eta(1+h), \qquad z=\zeta(1+h)$$

Hence

$$(\xi^2+\eta^2+\zeta^2)(1+h)^2(1+2q)-2p(1+h)^2\zeta^2=1$$

The values of h given in the table have to be multiplied by 10^{-5} in order to express them in terms of the Moon's radius ; we may therefore neglect their squares.

Doing this and remembering that

$$\xi^2+\eta^2+\zeta^2=1$$

we get

$$p\zeta^2-q-h=0 \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

The coordinates ξ, η tabulated in the catalogue apply to the point P, fig. 1 ; they must therefore be corrected by the addition of the quantities denoted in § 2, equations (3), (4), by $\delta\xi$, $\delta\eta$, and corresponding values of ζ^2 computed. Each of the thirty-eight points now gives a conditional equation of the type (6) for the determination of p, q.

These equations were formed and weighted by multiplying each by $\frac{1}{10^3\epsilon}$ where ϵ is the probable error of the corresponding value of h.

Normal equations were formed, and solved and the residuals from all the conditional equations computed.

The solution gave

$$p = +\cdot00043 \pm \cdot00030$$
$$q = +\cdot00078 \pm \cdot00022$$

But on applying Peirce's criterion it appeared that the equation due to *Purbach A* should be rejected. Its residual was

$10^{-5} \times 532$, whilst the criterion gave $10^{-5} \times 472$ for the limit. The rejection here seems entirely justifiable. The value found for the altitude may be erroneous, but the point certainly lies low, and the position found for it is so much below the mean surface given by the other points that it ought not to be employed in a determination of the mean surface of the Moon from a few points only.

Rejecting this point and solving again it was found that

$$p = +\cdot00052 \pm \cdot00027$$
$$q = +\cdot00079 \pm \cdot00019$$

and therefore

$$\frac{1-q+p}{1-q} = 1\cdot00052 \pm \cdot00027$$

The smallness of p and q justifies the neglect of the squares.

§ 7. *Provisional Determination of the Radius of the "Mean Sphere."*

If the result of this investigation is provisionally accepted it enables me to give greater definiteness to the "mean sphere" to which my points were referred, and in terms of whose radius their coordinates are expressed.

There is, so far as I know, no direct evidence that the observed disc of the Moon is sensibly elliptical. I therefore assume that the value here found for the polar radius may be taken as that of the radius of the whole disc. But it must be remembered that the radius I have taken passes through the crests of the craters. To determine what allowance should be made for this I took the mean of the heights of sixty-one of these small craters above the exterior level as measured by Schmidt. This was 0·45 mile. I therefore assume that the mean surface I have found lies 0·45 mile outside the mean surface of the Moon. This would increase the radius by $0''\cdot40$.

If we adopt L. Struve's value $15'\ 32''\cdot65$ for the radius of the disc, the radius of my mean sphere would then become 1·00079 of $(15'\ 32''\cdot65 + 0''\cdot40) = 15'\ 33''\cdot79$ at the mean distance of the Moon.

§ 8. *Discussion of the Result.*

Dr. Franz's value for the elongation of the Moon towards the Earth given in *Die Figur des Mondes*, which I have already referred to, is, I believe, the best hitherto published. The value is

$$+\cdot00114 \pm \cdot00390$$

The value now found is less than one-half of this, and the probable error less than one-fourteenth of that of the previous determination.

The determination now made applies only to a particular meridian, whilst Dr. Franz's points were distributed over the Moon. It is almost certainly possible to fit an ellipse on to a particular meridian with greater exactness than a spheroid could be fitted on to the whole surface, and this will probably account for part of the diminution in probable error. If sufficient material can be obtained, it will perhaps be well to attempt to determine the figure of the Moon by determining a number of sections in the way here attempted for the central meridian.

A part also of the improvement may be attributed to the fact that Dr. Franz's constants depend on measures of the limb. He himself attributes some of the discordances in his results to this cause, and in his subsequent work he seems to have altogether abandoned such measures.

My own experience, independently obtained, has been similar to his (*Monthly Notices R.A.S.* vol. lxii. p. 42).

It is interesting to note that the values of the moments of inertia of the Moon obtained by Dr. Franz in his discussion of the "Physical Libration" were such as would belong to a homogeneous ellipsoid with its principal axes in the ratios 1·0003 : 1 : ·9997, giving for the principal meridian an elongation ·0006, which agrees closely with that here found (*Die Figur des Mondes*, p. 2).

A difficulty in interpreting the present result arises when we notice that $1-q+p<1$, or that the longest radius of the Moon is less than that of my mean sphere. This might be explained by supposing that the craters used as standard points had on the whole higher walls than the thirty-eight here considered. But it was found in § 3 that the mean altitude of the fourteen points measured by Dr. Franz and myself was 5×10^{-5} of the Moon's radius above my mean sphere, so that the mean sphere would seem to pass pretty nearly through these points. And, moreover, the amount of this defect from unity is only 27×10^{-5} of the Moon's radius, which is just equal to its probable error. The true explanation is, I think, that the accidental irregularities of the surface are considerably greater than the elongation.

The comparison of the altitudes of the same fourteen points with those found by Dr. Franz seemed to indicate that the radius of his mean sphere was 170×10^{-5} of the Moon's radius less than mine, which would make it less than the least radius of the figure now determined for the Moon; but I hope that many of these small discordances will be considerably modified when we have better determinations of the altitudes.

The smallness of the elongation now found partly disposes of

a difficulty I had felt in combining the measures made on different plates. It was impossible to use the same set of standard points for all the plates : those illuminated in one were beyond the terminator in another, and there was therefore no guarantee that the measures were all referred to the same mean sphere. Had the elongation been considerable the radius of the mean sphere would have been sensibly affected by the distances of the standard points from the mean centre of the disc, and a further correction would have been necessary.

The means of these distances are not very different for the different plates, and I do not think that any sensible error is introduced in this way. It may become more important in future to consider the actual heights of the walls of the different craters and the general level of the part of the surface on which they are situated.

The general result of the inquiry would seem to justify the hope with which it was undertaken, and to show that the method is one of considerable promise. But, in spite of the reduction in the probable error, it is not safe to place too much reliance on the numerical results here obtained. A glance down the figures in the altitude column of the table in § 3 will show that while the general level of the surface falls as the central meridian crosses the Mare Imbrium, it rises again in the neighbourhood of *Anaxagoras* ; a conclusion which may be verified by an inspection of the photographs. And it is obvious also that a very different ellipse would have been obtained had the last two points on the list been omitted. In fact, if the equations are solved without these points we obtain

$$p = +\cdot00123 \pm \cdot00029$$
$$q = +\cdot00136 \pm \cdot00022$$

the elongation being more than doubled and almost exactly coinciding with that obtained by Dr. Franz. The change hardly affects the length of the radius pointing directly towards the Earth whose value is $1-q+p$, which in the first case is ·99973, and in the second ·99988 ; but the value of the polar radius $1-q$ falls from ·99921 to ·99864.

There can, I think, hardly be a question that, so far as can be determined from the present investigation, the result given in § 6 more nearly represents the true figure of the Moon. The elevated region in which *Anaxagoras* lies is not an isolated or exceptional feature, and it would not be right to attempt to bring the mean surface down to the level of the Mare Imbrium to the exclusion of this higher land. But the fact that the omission of these two points so completely alters the character of the solution shows the necessity for further investigation and for the inclusion of a greater number of points.

I have discussed this question at some length because a great

part of the work has been accomplished by the help of grants received from the Government Grant Committee of the Royal Society, to whom I wish here to express my gratitude. If it is to be continued on the same scale it can only be by means of further assistance from the same source, and it seemed very desirable that an effort should be made at the earliest possible opportunity to ascertain to what extent the investigation is likely to increase our knowledge of the Moon.

There is another reason why the attempt should be made to determine the individual altitudes with all possible precision. It has been frequently suggested that measures of well-defined points upon the Moon's surface should be made with meridian instruments, instead of measures of the limb, in order to determine the position of the Moon. In order to compute the apparent position of a point on the surface relatively to the centre of the disc at any given instant it is necessary to know its selenographical coordinates in all three dimensions if full advantage is to be obtained of the increased accuracy of which I believe the method to be capable.

The same knowledge will be required if we are to adopt photographic methods of determining the Moon's position as has been proposed by Professor Turner (*Monthly Notices R.A.S.* vol. lxiv. p. 19).

The Spectroheliograph of the Solar Physics Observatory.

By William J. S. Lockyer, M.A., Ph.D., F.R.A.S.

Introduction.

Since the year 1897 numerous experiments have been made at this observatory to determine the best design for a spectroheliograph based on the Hale-Deslandres principle. The improvised instruments took many forms during the course of the trials, until finally, in 1901, the definite form resulting from these experiments was decided upon and the instrument purchased.

In the present paper it is proposed to describe somewhat in detail the instrument now at work, and give a brief account of some of the first results which have been obtained during the past year.

The principle of the spectroheliograph may first be described briefly. Imagine an ordinary student's spectroscope with a collimator and observing telescope, but with the addition of a plane mirror in the optical train to render the collimator and telescope parallel to each other. Replace the eyepiece of the observing telescope by a slit (secondary), thus providing a means of isolating any small portion of the spectrum. If now an image of the Sun fall on the slit (primary) forming part of the

collimator, then the light passing through the secondary slit will only consist of a small part of the spectrum corresponding to the slit's width. By adjusting the secondary slit so that only one line in the spectrum, such as H_β, is allowed to pass through it, we shall have an image in hydrogen light of that part of the solar image that is passing between the jaws of the primary slit.

By moving the primary slit, and with it the whole spectroscope, across the solar image, different strips of the latter pass consecutively through the collimator, and consequently corresponding strips in hydrogen light will find their way through the secondary slit. If the solar image be kept stationary, a fixed photographic plate placed nearly in contact with the secondary slit, and the whole spectroscope moved parallel to the plane of the slit plates and in an east-and-west direction horizontally, a picture of the Sun is built up in hydrogen light on the sensitive plate.

The form of instrument in which the solar image and photographic plate are kept stationary is perhaps almost the ideal. There is no restriction to the size or weight of the spectro-heliograph, for the only motion required is that in a horizontal direction, and this can be secured without great difficulty.

In their account of the Rumford spectroheliograph Hale and Ellerman * pointed out that it was impossible to use an instrument of this kind with the 40-inch refractor, since the great weight of the moving parts would have set the telescope in vibration. In place of this the principle of moving the Sun's image across the primary slit by a uniform motion of the telescope tube in right ascension was adopted, the photographic plate being moved at the same time and rate across the secondary slit. In the Potsdam spectroheliograph, a brief description of which has recently appeared,† the instrument is mounted on a Grubb refractor, and the collimator and camera tubes are moved against a fixed solar image and photographic plate.

In all the preliminary experiments made at the Kensington observatory to determine the most suitable form of instrument, that employing a fixed solar image and photographic plate was always used and finally adopted ; the present instrument has been designed on these lines.

The complete instrument as now erected consists of three parts :

1. A siderostat to throw the solar beam in a southerly and horizontal direction.

2. A lens placed in this beam to form the solar image.

3. The spectroheliograph to photograph in monochromatic light the image thus formed.

* *Publication of the Yerkes Observatory*, vol. iii. pt. 1, 1903, p. 6.

† *Astrophysical Journal*, vol. xxi. No. 1, 1905 January, p. 49.

The Siderostat.

This instrument does not differ in the main from an ordinary Foucault's siderostat except in point of size. It was constructed by Messrs. T. Cooke & Sons, York.

The massive base (6 feet $3\frac{1}{2}$ inches $\times$ 3 feet 1 inch) is supported on three large screw-heads each of which rests on a circular iron plate embedded in concrete. On the north side an antagonistic screw arrangement is attached to allow of an adjustment of the base in azimuth. The highest portion of the instrument is 7 feet $4\frac{1}{2}$ inches from the surface of these plates, while the centre of the horizontal axis on which the mirror moves is 4 feet $5\frac{3}{8}$ inches from the same level. The mirror itself is 18 inches in diameter and $2\frac{7}{8}$ inches thick, and was ground by the late Dr. Common. The mirror, cell, and projecting arm are counterbalanced by two levers carrying weights the ends of which are fitted at the back of the cell, two points in the mirror fork being used as a fulcrum for each. The driving clock is of the ordinary Cooke construction, and the weights are suspended over strong wooden gallows fitted close to the north side of the instrument. The clock is fitted with a Russell control, and this is operated from the spectroheliograph room by an electric pendulum made by Sir Howard Grubb. A four-volt cell is used to drive the pendulum, while two cells of four volts each supply the sparking current which operates the controlling armature.

The same two cells are also employed for actuating the two small motors connected by tooth-wheeled gear with the right ascension and declination axes for supplying slow motions in these two coordinates. The switchboard for this is situated in a convenient position on the concrete pillar supporting the spectroheliograph plate-carrier, together with the north foot of the fixed triangular framework, in order that the observer at the secondary slit can adjust the solar image in relation to the primary slit.

To inform the observer when the siderostat driving clock requires re-winding, an automatic electric arrangement rings a warning bell in the spectroheliograph house.

The instrument is sheltered in a wooden house the upper portion of which is on wheels, and can be moved on rails towards the north when necessary.

The Object-glass.

When the instrument was first set up a 6-inch Taylor lens of 20 feet focal length was employed, but this has been replaced by a 12-inch photo-visual Taylor objective having a focal length of 18 feet. It is mounted in a metal cell which is carried by a stout vertical mahogany frame fitted with a vertical adjustment for raising or lowering the object-glass, and adjustable in a horizontal direction for focussing purposes. This frame forms

part of a strong base resting on a concrete pillar. The object-glass lies to the south of the siderostat at a distance of 40 feet 4 inches, and forms an image of the Sun of $2\frac{1}{5}$ inches diameter on the primary slit plate.

The Spectroheliograph.

The Slits.—The primary slit was made by Messrs. Ottway & Co., Ealing (Plate 9, fig. 1). It is constructed of brass, and has straight jaws of platinum-iridium 3 inches in length. These jaws are mounted on two small arms pivoted at their middle points so that the milled-headed screw by which the width of the slit is varied operates both jaws simultaneously. The slit plate is mounted at one end of a brass tube, and is capable of rotation in a vertical plane; there is, further, an adjustment for focussing by which this tube can be moved in and out of the collimator tube, the different positions being read off by an attached scale. Behind the slit, but operated from outside the brass tube to which it is attached, is a small exposing shutter. On the slit plate itself a temporary artificial dust mark has been improvised consisting of a needle the point of which is easily adjusted by means of a screw to just cover a small part of the middle of the open slit. The object of this is to give a datum line in the resulting picture for purposes of orientation of the solar disc.

To obtain photographs of the chromosphere and prominences a zinc disc, shown in position in Plate 9, fig. 1, a trifle smaller than the solar image, is placed close up to but not touching the slit plate, and retained there by a thick metallic wire arm rigidly secured to a small wooden block sliding in another block with V grooves. This latter block is capable of rotation in a horizontal plane, but otherwise firmly fixed to a support resting on the same concrete base as that on which the plate-carrier rests. The disc is placed friction tight on to the wire, so that other discs of various sizes may be substituted should occasion occur. During exposure this disc becomes exceedingly hot, so that it must be made of metal and either riveted or fixed on its support friction tight. To allow for the expansion of the metal the solar beam is allowed to fall on the disc some minutes before the position of the image is finally adjusted.

The secondary slit (Plate 9, fig. 2) is somewhat more elaborate than the primary. It was constructed by Messrs. Hilger. It is constructed mainly of gunmetal and brass, with the exception of the slit plates, which are of non-corrodible metal. It is supplied with four adjustments in a vertical plane—namely, a movement of rotation, an adjustment in the vertical and horizontal directions, and the movement of one jaw relatively to the other. For focussing purposes it is capable also of an adjustment bodily in a direction at right angles to the slit plates.

In order to be able to utilise at some future time the instru-

ment for photographing sun-spot spectra when a large grating becomes available, the two slit plates with the metal slides can be removed entirely from the metal-carrier, and easily replaced afterwards, without deranging any of the previous adjustments of the position of the slit relative to any particular spectrum line.

The jaws of the slit are $3\frac{1}{2}$ inches long and curved to a radius (48·38 inches), corresponding to the curvature of the "K" line of calcium.

Professor Hale, in the description of the Rumford spectro-heliograph, adopted the method of dividing the curvature equally between the primary and secondary slits. A careful examination of the images formed by the South Kensington instrument has shown that any distortion which might arise from the method there in use is so inappreciable that it may practically be considered to be non-existent.

In order to allow the photographic plate to be placed nearly in contact with the slit plates, and clear of all projections, every part of the mounting and screw adjustments of the latter are retained on the tube side of the plane of these plates.

The Plate-carrier.

The plate-carrier, which was made by Messrs. Watson & Sons, consists of a vertical framework of mahogany carrying a second framework of similar wood sliding in grooves and capable of adjustment in a vertical direction, the clamping being performed by means of a milled nut (Plate 9, fig. 1). The whole of this framework has a strong mahogany base fitted with three levelling screws, resting on another mahogany base connected by rack and pinion to a heavily weighted box placed on a concrete pillar. The dark slides, made of mahogany and aluminium, are inserted horizontally in grooves on the vertical framework.

The draw slide is of thin aluminium, and fits in aluminium grooves. The inner portion of the dark slide is specially constructed so that the film side of the photographic plates is as close to the slide as possible. This is imperative, since the jaws of the secondary slit and the film of the photographic plate must be placed as near together as is practically possible.

The Optical Parts.

The two slits to which reference has just been made are mounted at one end of a double tube 6 feet long; each tube is rectangular in section and made of mahogany. At the opposite end are placed two 4-inch photo-visual Taylor lenses, each of 6 feet focal length and fixed permanently, the adjustment for focus being made at the slits, both of which are movable in and out of the tubes. In the optical axis of the collimator, but $16\frac{1}{2}$ inches to the south of the collimating objective, is placed a

6-inch plane mirror (Plate 9, fig. 3) held in a brass frame supported on three levelling screws. To preserve the surface when the instrument is out of use a glass cover is placed over the mirror without disturbing any of the adjustments. Close to the camera lens is a 6-inch Henry prism of 45° angle supported on a wooden base, also on levelling screws. The whole of this end is inclosed in a blackened cardboard box, which is easily removed when necessary.

In this way the light which falls on the primary slit is rendered parallel by passing through the collimating lens; it then falls on the plane mirror, is reflected at an angle on to the surface of the prism, and after passing through the prism falls on the object-glass of the camera, the spectrum being formed in the plane of the secondary slit. Thus the total deviation of the beam for "K" is 180°.

To avoid internal reflexions diaphragms are fixed both within the collimator and camera tubes, and others are placed on the baseboard carrying the prism and reflector. The length of spectrum between F and K is 1·62 inch, so that the dispersion is sufficient if only the "H" and "K" lines of calcium be employed. For work with other lines and for spot spectra it is proposed to replace the reflector by a grating, but up to the present time it has not been possible to obtain one of the required size.

In one of the accompanying figures (Plate 9, fig. 4) is given a reproduction (enlarged twice) of the "H" and "K" region of the spectrum, showing the curvature of the lines. A spot region had been previously adjusted on the primary slit, so that the dark band crossing the spectrum is due to the spot absorption. Reversals of the "H" and "K" lines may also be seen above and below the darkest portion of this band.

The Moving Platform.

The satisfactory working of the spectroheliograph depends largely on the efficiency of the moving platform on which the optical parts are placed (Plate 10). This part of the instrument was constructed by the Cambridge Scientific Instrument Company, and has given great satisfaction.

It consists, in the first place, of a triangular iron framework resting at its three corners on levelling screws, each of which is supported on a concrete pillar. The long side of this triangle rests on levelling screws lying in a north-and-south direction; the third foot is situated towards the west. On the upper sides of this triangle, and near each of the angles, is fixed a small plane-surface steel plate adjustable on the framework by means of four screws. On each of these plates a small steel ball, with a diameter of one inch, is free to move within prescribed limits in any direction.

A similar triangular framework, but with the three similar

adjustable steel plates fixed to its lower surface, is placed on the top of the first framework, the three balls running between the surfaces of the corresponding steel plates at the corners. To control the direction of motion of the upper framework a guide-bar is fixed on the lower one lying horizontally east and west. Against this the upper framework, which is fitted with two metal struts projecting downwards, is pressed against the guide-bar by two small levers, each having a roller on one arm running on the opposite side of the guide-bar and a small weight suspended on the other arm. These weights keep the metal projections in contact with the guide-bar. The motion of the upper framework is obtained by means of a steel band, one end of which is fixed to the west corner of this framework, while the other, after passing over a pulley fixed to the lower framework, carries a set of weights. To regulate this motion from east to west the upper framework is fitted with a metal plunger which projects downwards and fits into a slot forming part of a piston in the regulating oil cylinder firmly supported on the lower framework. This piston consists of a hollow cylinder open at one end and at the other two apertures, one comparatively large, with a valve to allow the entry only of the oil from the larger cylinder; and the other, variable in size, for the outlet of the oil from the piston during the movement of the framework. The open end of this cylindrical piston moves in and out of a closely fitting cylinder closed at the other end, so that the oil which is inclosed in them can only find its exit through the opening of variable aperture. A milled-headed micrometer screw and scale moving with the slot into which the plunger fits and attached to the piston varies the size of this oil outlet and allows perfect control of the escape of the oil from the inside of the piston to the cylinder without. For the purpose of setting the framework into its starting position, it is pushed towards the east by means of a long screw with attached handle working in a thread fixed in the same upright that carries the pulley. When the starting position has been reached, a small catch-lever fitted to one end of the piston falls into a slot on a projection inside the oil cylinder. This holds the framework in position, so that the setting screw is freed and can be wound back again. The actual starting of the motion is operated by a starting-handle attached to a long metal rod passing through the upper framework which raises the catch-lever out of its slot, and thus releases the piston. The framework thus being set free the weights pull it over in a westerly direction, and the piston pushes the oil from one cylinder to the other through the orifice the aperture of which is controlled by the adjusting micrometer screw. The maximum length of run which can be obtained by the motion is four inches.

With regard to the actual quality of the movement of the upper in relation to the lower framework, the motion is extremely smooth and far exceeds expectations. Even with such rapid movements as that of $2\frac{1}{8}$ inches in fifteen seconds scarcely any

trace of uneven motion can be detected, even when the negative has been enlarged four diameters.

The exposures, varying within wide limits, demanded by the changing clearness and brightness of the image on the slit require fairly accurate settings of the micrometer screw for any particular exposure. Experience soon showed that temperature greatly influenced the rate of escape of the oil (sperm) which rate determines the "run." A chart giving working details of the influence of temperature became an immediate necessity. Preliminary observations during adjustment gave some data which put in the form of a table aided in setting ; the daily results being added, the table gradually became of greater extent, accuracy, and value. At present a setting of sufficient closeness by means of the chart is thus readily made.

The table consists of a sheet divided into squares : in these the actual times of runs are entered, their position in the table being determined by a vertical scale of temperature and a horizontal scale of micrometer readings. With a full sheet the conditions for a run of any duration can be seen at once. As each square accumulates data, corresponding to a definite temperature and micrometer reading, differences are noticed. These are no greater than might be expected considering the difficulty of timing a passage of the slit, the alteration of focal length, and consequent change of size of the primary image, and the real seasonal change of semi-diameter. The temperature, though taken close to the clepsydra, cannot with certainty be said to represent that of the oil.

The Optical Parts in Relation to the Moving Platform.

The double tube is placed parallel to the long side of the triangular frame, this having previously been set in a north-and-south line. The tube is strongly screwed down on to the framework in such a way that the northern end carrying the two slits overhangs it.

To the south of the collimator and camera objectives is a strong base board also firmly fixed to the framework (Plate 9, fig. 3), and on this are placed the levelling-stands carrying the 6-inch reflector and prism.

The mean position of the optical axis of the collimator tube which carries the primary slit is so adjusted that the latter bisects the solar image formed in the meridian line by the 12-inch objective. In this way the slit can be moved to either side of the solar image to a maximum distance of eleven-twelfths of an inch, and a clear run of the slit across the image, which has a diameter of $2\frac{1}{6}$ inches, can be secured. For obtaining photographs of the prominences on the solar limb this clearness is found to be ample.

Adjustment of Secondary Slit on the "K" Line.

Up to the present time the setting of the central portion of the "K" line on the secondary slit has been accomplished by visual observation with the aid of a watchmaker's black eye, but it is hoped that a more satisfactory method will be available when the alterations and additions now in progress are completed. On bright days and with a high sun there is very little uncertainty about setting the line correctly, especially if a spot region be adjusted on the primary slit, so as to produce reversals on the "K" line at the secondary slit. On dull days the "K" region of the spectrum is very faint, and correct setting occupies much time. To control the correctness of the adjustment a photograph with the secondary slit wide open is often taken. This photograph also serves several useful purposes, for from it not only can the verticality of the secondary slit in relation to the spectrum lines be at once observed, but the position of the photographic plate in relation to the focal plane can be checked.

The Taking of the Photograph.

The adjustment of the secondary slit on the "K" reversal being satisfactory it is closed down by a micrometer screw to the desired width. Putting the primary slit in the meridian line through the siderostat and 12-inch lens, the solar image is brought by the siderostat slow motions into the meridian and centred on the slit. The focussing of the image on the slit is accomplished by placing a thin sheet of paper on the slit plate and moving the 12-inch lens until the solar limb, or spot, is quite sharp. In consequence of the brilliancy of the image the observation has to be made through tinted glass. The small shutter behind the primary slit is then closed. The loaded plate-holder is next slid into the carrier, and both secondary slit and carrier are as securely wrapped in velvet as is consistent with the necessary freedom of relative movement during the exposure. Pushing the whole upper framework, and with it the primary slit, to the east of the stationary solar image by the screw with attached handle, described above, the instrument is ready for a "run." The brightness of the image now determines the length of the exposure to be given.

The approximate temperature of the oil in the clepsydra is read by the attached thermometer. This enables the necessary micrometer screw-reading for the length of run to be obtained from the table of data compiled from previous observations.

The screw-plunger adjustment, facilitated by a conveniently placed electric pea lamp which lights the scale and a reading lens, both carried by the upper triangle, is readily made.

The slide of the plate-holder is now withdrawn under its velvet cover, and the signal for a favourable opportunity for

exposing waited for. This given, the final operations of opening the shutter of the primary slit and releasing the starting-handle are performed. The primary slit then moves uniformly westward, touches and glides through the stationary solar image, the "K" light record being continuously built up on the stationary photographic plate.

The time of transit of the slit across the image is indexed as the "run." Closing the slit shutter and the slide of the plate-holder completes the essential instrumental operations.

To obtain a photograph of the prominences on the limb, a longer "run" and the eclipse of the Sun's image by a zinc disc (described above) are necessary.

When these limb photographs are taken it is always attempted to produce a "composite picture" by making an exposure for the disc on the same plate. For the limb picture the disc must be placed vertically and close to, but not touching, the primary slit. Its size is so chosen that it only allows an extremely small portion of the solar image at the limb to fall on the slit plate. Being set in position and the micrometer screw adjusted for the longer "run," the procedure is the same as before.

To maintain the image in position during the necessarily much longer exposure, the siderostat slow motions are used when necessary. After the completion of the exposure for the limb the primary shutter is closed and the instrument is wound over into the initial position. The micrometer screw is then altered to reduce the exposure to that required for the disc picture. The metal disc is then removed, the primary exposure shutter opened, and the slit is allowed to traverse the whole solar image, producing the ordinary "K" disc picture.

The developed result is thus a composite picture in "K" light of prominences at the limb and surface markings.

After many trials it has been found that a "limb" photograph requires on the average sixty times the exposure of that necessary for the "disc" pictures. The plates, which have given the greatest satisfaction throughout, have been the Imperial Ordinary, of the Imperial Dry Plate Company, developed with hydroquinone.

Before proceeding to describe the results obtained, mention may be made of the difficulties that have arisen regarding the use of Taylor lenses. In the first place the focal length of the 12-inch objective is always undergoing alterations from minute to minute, depending on temperature. Thus, for instance, the focal length as determined by a star at night is shorter than that measured by the Sun during the day time.

The tendency is for the focal length to increase for a rise in temperature, but the changes do not vary regularly with regard to the temperature of the room. The presence or absence of the solar beam on the objective, or the passing of a cloud which cuts off the sunlight, are sufficient to cause the focal length to change considerably.

Great care has therefore to be taken to allow the solar beam to fall on the lens for some minutes before taking a photograph and to adjust the focus on the primary slit immediately before making the exposure.

In the case of the limb photographs, when exposures of some minutes are required, this variation is a source of much trouble, and responsible for many unsatisfactory results.

Again the 4-inch Taylor objectives forming parts of the collimator and camera tubes have also been found to vary their focal lengths, though not to such a great extent. This has necessitated a frequent check being made on the position of the focal plane at the secondary slit.

Results.

Up to the present time the instrument has been employed for securing two classes of routine photographs.

(1) Disc photographs in "K" light.

(2) Composite disc and limb photographs also in "K" light.

These latter, though always striven for, were only obtained on the days of more continuous clear weather, and often several were obtained on days of special interest, such as when spots were near the limb.

Examples of these two kinds of pictures are shown in Plate 11, figs. 1 and 2, the former having been taken on 1904 September 20, and the latter on July 19 of the same year.

Dealing in the first instance with the photographs showing the "K" markings on the disc, the following general remarks, gathered from a preliminary examination of all the best plates taken between April and November, may be of interest.

The general feature of the surface of the disc is a universal "mottling" which seems to be made up of a fine mesh of small branching lines. They are not restricted to any zone, but are of the same character in both polar and equatorial regions. On clear days, with good definition and accurate setting of the line on the slit, this mottling becomes very obvious. More conspicuous than the above are the bright and more or less compact patches in middle and low latitudes. They have the appearance of being made up of the mottling, but exaggerated in size and intensity, and the compactness appears to be produced by the filling up of the interspaces with bright patches; in fact in these regions there is a tendency to form a more intense nucleus with radiating arms, and the resemblance to foam traces in deep water disturbed from below is very striking. Plate 12 gives an idea of this apparent building up of a flocculus in the manner above described. It will be seen that there are very frequently long streaky bright portions springing from a central nucleus and having subsidiary ramifications. A three-legged formation with central brightening is a very common type of structure.

In the majority of cases flocculi exist where no spot is obvious, but spots never appear unless surrounded by these calcium clouds. They resemble and possibly are identical with (in part at least) the faculæ, and in consequence are restricted to the zones of the latter.

When a number of groups appear on the disc together the arrangement into equatorial belts is very pronounced. Their distribution in longitude is intermittent more than continuous (see Plate 11), but this may not be the case at other phases of the sun-spot cycle.

With regard to the relation of spots to these calcium clouds it is noticed that the former appear more generally near the head of or precede the apparently trailing masses of the latter with respect to the solar rotation. Typical examples of this relationship are given in Plate 13. There seems to be little doubt that the position of spots in flocculi is not due to chance, but that there is some physical reason dominating the cause and effect in their relative positions. This is a point that requires investigation when several complete series of photographs, extending over semi-rotation periods of the Sun, have been secured. Such photographs will probably throw much light on the various phases in the life-history of a spot group, and classifications, such as that put forward by Father Cortie, S.J.,* may possibly be extended and explained.

Turning now to the results gathered from an examination of the composite pictures showing the disc and limb taken at two consecutive exposures on one plate—an example of which is illustrated in Plate 11, fig. 2—the following are the chief points noticed :

Prominences in all latitudes, even near the solar poles, give strong images in "K" light.

The frequent occurrence of prominences outside the zone of flocculi during this period did not apparently alter in any way the regular mottling on the disc in those latitudes.

When an intense flocculus was on or near the limb it was not always accompanied by a large prominence at the same position angle or even in its neighbourhood. The impression gathered from a series of photographs taken especially for this purpose was that prominences were generally independent of flocculi, and *vice versa*.

On continuously fine days, when several composite photographs could be secured at intervals, an opportunity was afforded of studying the changes in the form of prominences from hour to hour.

Two examples of such changes are here described and illustrated in Plate 14, figs. 1 and 2. Those in fig. 1 were taken on 1904 July 14 at $11^h\ 8^m$ A.M. and $12^h\ 8^m$ P.M. G.M.T. It will be noticed that during the interval of time between the

* *Astrophysical Journal*, vol. xiii., 1901, p. 260.

two photographs, about one hour, a startling change occurred to the largest prominence ; and not only has its height been considerably increased, but its form has altered. The material forming the prominence seems to have been ejected from the chromosphere and then to have met a strong current moving polewards (*i.e.* from left to right in the figure), which has thrown this material in that direction. The change of height from 50,000 to 60,000 miles in this interval gives an approximate rate of movement away from the chromosphere of about two and a half miles per second.

It will be noticed that the other large prominence (No. 5) in the same figure has during that interval nearly disappeared in spite of its exceeding intensity in the first photograph. In the second case (Plate 14, fig. 2) we have an enormous prominence in the spot zone that was secured on 1904 July 19, the upper and the lower photographs having been taken at $11^h\ 44^m$ A.M. and $3^h\ 52^m$ P.M. G.M.T. respectively. This prominence was situated in the S.E. quadrant, the higher portions being directed towards the equator. The approximate greatest heights and lengths on the two photographs were as follows :

h m	Length.	Height.
11 44	192,000	55,000
3 52	216,000	60,000

[To show the complete record both on the limb and disc at the first of these times, see Plate 11, fig. 2.]

Although the interval here is over four hours, there is not such a considerable alteration in form as that recorded in Plate 14, fig. 1.

In many of the remarks relating to the disc and limb pictures it must not be forgotten that they only apply to the photographs secured in 1904. At any other time, such as a sun-spot minimum or maximum, the solar activity is different, and therefore changes of another kind may be apparent. Thus, for instance, the mottling on the disc may be more, or it may be less, pronounced than it was in 1904, while the flocculi may be more connected in longitude than was the case in 1904, when they were intermittently distributed in longitude.

In conclusion, I should like to express my obligations to the Director of the Observatory, Sir Norman Lockyer, for allowing me to prepare the present paper for communication to the Society. Messrs. W. Moss and T. F. Connolly assisted me in taking the photographs, and Mr. J. P. Wilkie in preparing the plates.

Description of the Plates.

PLATE 9.

FIG. 1.—The north ends of the collimator and camera tubes showing the two slits, the plate holder carrier and metal disc in position for a limb photograph (p. 476).

FIG. 2.—A view of the curved secondary slit showing the screw adjustments (p. 476).

FIG. 3.—The south ends of the collimator and camera showing the arrangement of the reflector and prism (p. 478).

FIG. 4.—The "H" and "K" portion of the solar spectrum (enlarged twice) illustrating the curvature of the lines. The sun-spot absorption is also shown and the reversals of the "H" and "K" lines near the spot region (p. 478).

PLATE 10.

General view of the spectroheliograph (p. 478).

PLATE 11.

FIG. 1.—Sun's disc in "K" light as photographed on 1904 September 20 at $10^h\ 42^m$ A.M. G.M.T. Exposure 73*s*. Enlarged about $1\frac{1}{2}$ time (p. 483).

FIG. 2.—Composite photographs of Sun's disc and limb in "K" light as photographed on 1904 July 19 at $11^h\ 45^m$ A.M. G.M.T. Exposure for disc 18*s*, and for limb 18*m*. Enlarged about $1\frac{1}{2}$ time (p. 483).

PLATE 12.

Flocculus region to illustrate how these large "K" clouds are built up by the amalgamation of the smaller but somewhat intense "mottling." The dates on the right-hand side correspond to the days on which the photographs were taken. The east limb of the Sun is towards the right. Enlarged about four times (p. 483).

PLATE 13.

Typical cases of spots preceding the main flocculus regions in the direction of the solar rotation. The dates on the right-hand side correspond to the days on which the photographs were secured. The east limb of the Sun is on the right-hand side in each case. Enlarged about four times (p. 484).

PLATE 14.

Enlarged photographs of "K" prominences illustrating changes after brief intervals of time.

FIG. 1.—Negatives taken 1904 July 14. The upper photograph was secured at $11^h\ 5^m$ and the lower one at $12^h\ 30^m$ P.M. G.M.T.

FIG. 2.—Negatives taken 1904 July 19. The upper and lower photographs were taken at $11^h\ 45^m$ A.M. and $3^h\ 59^m$ P.M. G.M.T. respectively. Enlarged about four times (pp. 484, 485).

K. H.

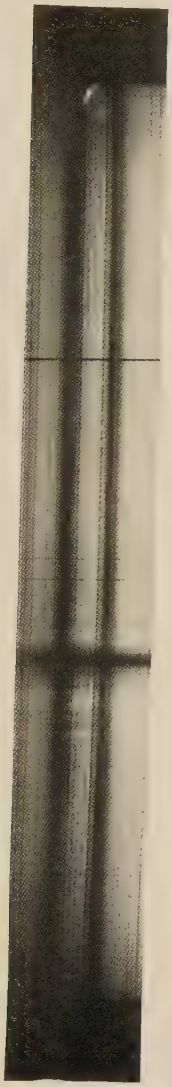


FIG. 4.

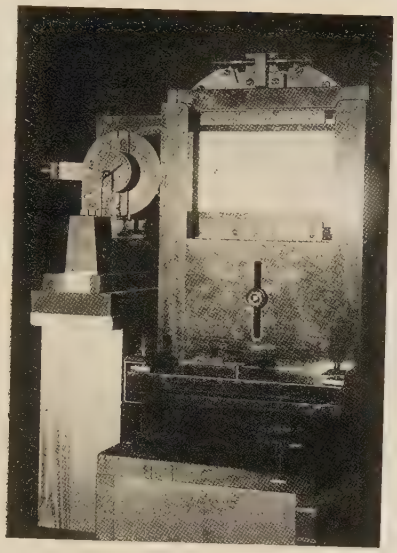


FIG. 1.

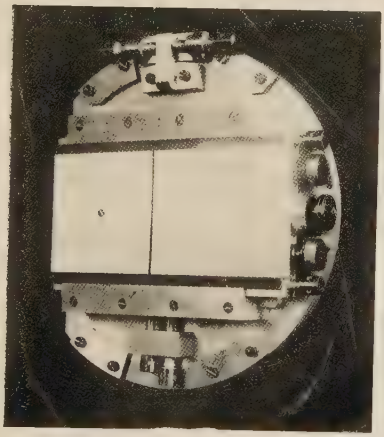
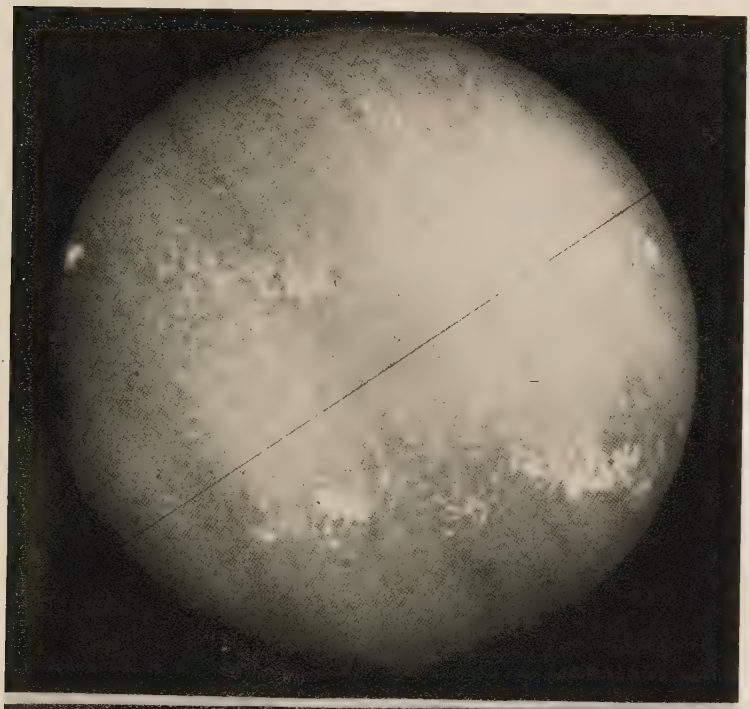


FIG. 2.

FIG. 3.

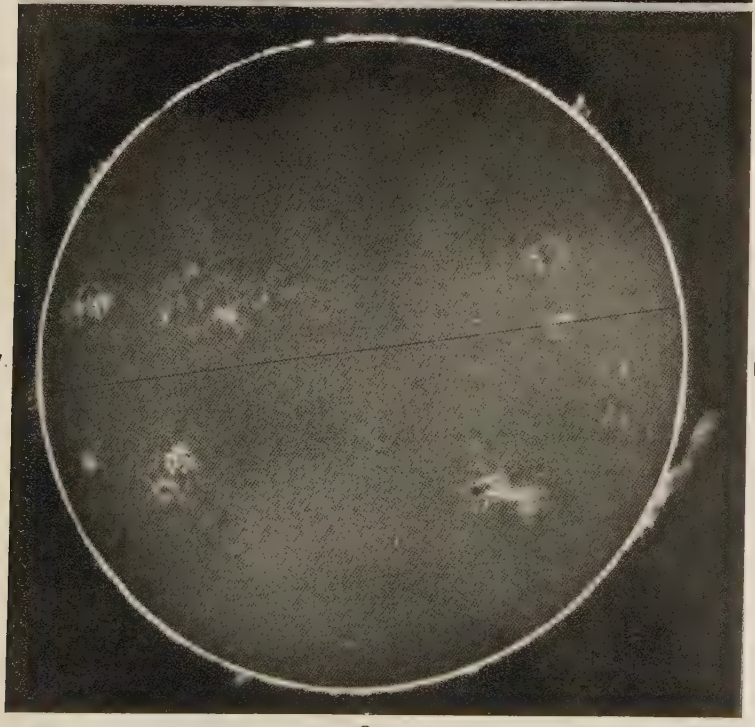
GENERAL VIEW OF SPECTROHELIOGRAPH.

N.



W. E.

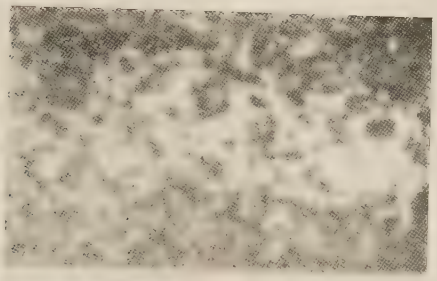
Fig. 1.



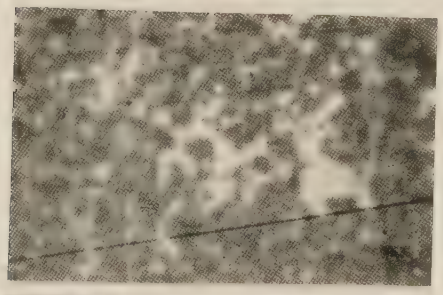
W. E.

Fig. 2.

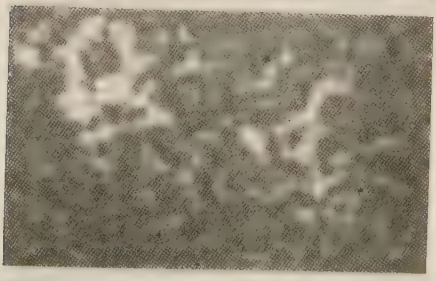
S.



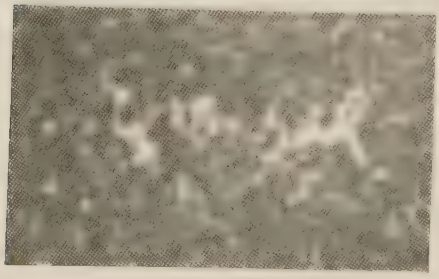
1904.
May 23.



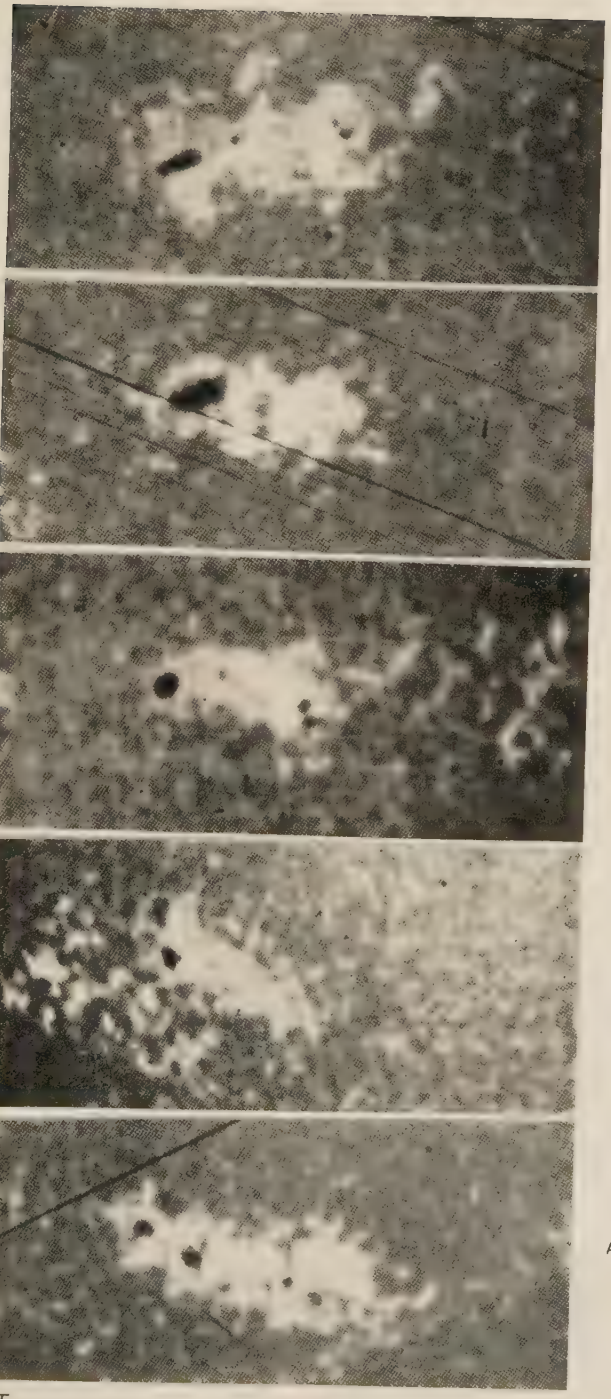
Aug. 2.



Sept. 26.



Sept. 26



1904.
April 27.

April 27.

July 14.

Aug. 2.

Aug. 29

WEST.

EAST.

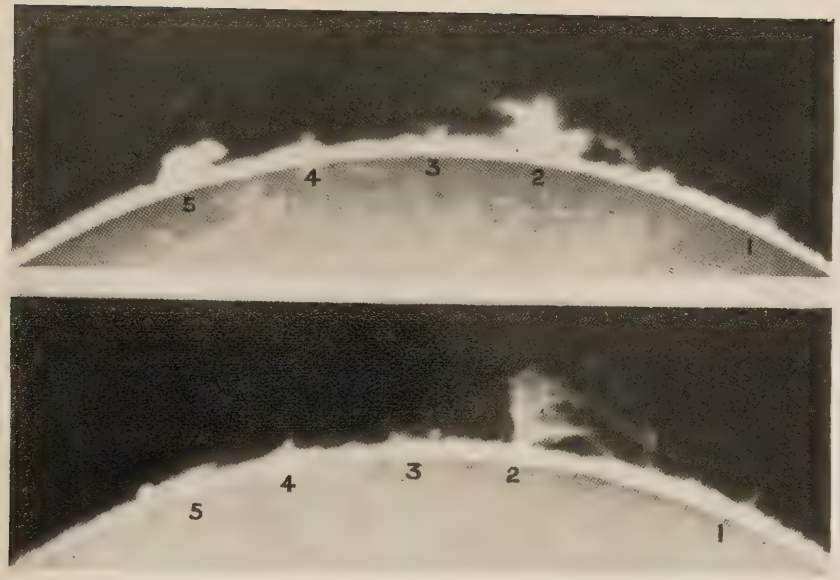


Fig. 1.

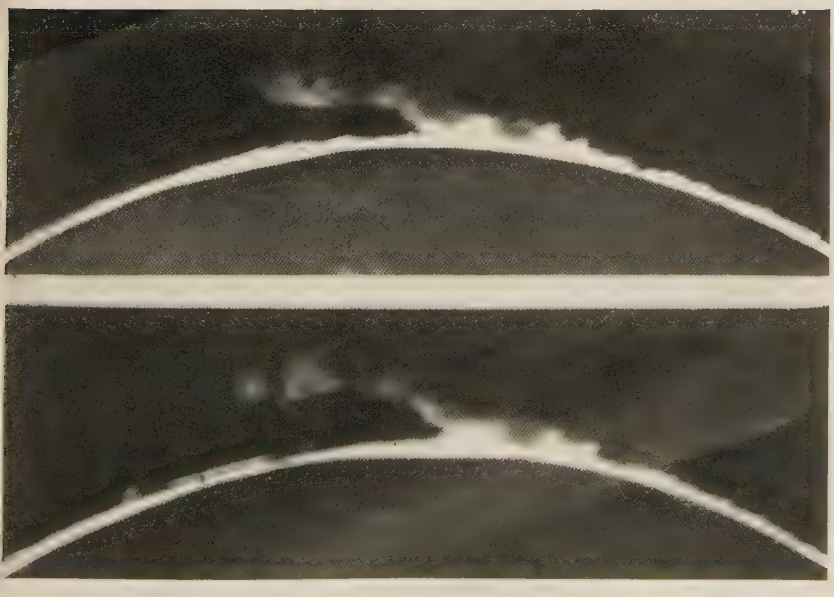


Fig. 2.

Notes on the Cœlostat and Siderostat. By H. C. Plummer, M.A.

1. At the present time the desirability of using telescopes of exceedingly great focal length is urgently felt. There is no insuperable difficulty in making reflectors or lenses with focal lengths of a hundred feet or more, but it is practically impossible to mount instruments of this size in such a way that they can be directed to any part of the sky and made to follow the diurnal motion of the stars. They must be placed in stationary positions, and the simplest construction is possible when the telescope is horizontal. In order that any part of the sky may be observed in the telescope, it is necessary to place a plane mirror in front of the reflector or object-glass. The mirror may be mounted in such a way that as a particular star describes its diurnal path, its image remains fixed in the field of the telescope. If this is the case the motion of the mirror must be of a simple and definite geometrical type (cf. *Monthly Notices*, vol. lxi. p. 459) which may be called siderostatic, although the motion of the siderostat may be produced by many different forms of mechanism. The use of a siderostat, notwithstanding the advantages which it offers, is not free from objection. In spite of the ingenuity which has been devoted to several forms of construction it has been generally felt that the mechanical design of the instrument leaves much to be desired. The largest and most elaborate siderostat yet made was that shown at the Paris Exhibition, 1900; but unfortunately no information seems to have been published on which a reliable judgment of its performance can be based. Dr. Johnstone Stoney (*Monthly Notices*, vol. lvi. p. 452) has suggested a way in which the chief mechanical defect of the older forms may be avoided, but I am not aware that the suggestion has been embodied in an instrument of sufficient size to test its practical value.

2. In addition to the admitted defects of the siderostat itself, a further difficulty arises from the variable rotation of the field. For photographic purposes the rotation may be compensated by one of several devices which have been suggested for turning the plate. But however complete the mechanical arrangements there will be the same ultimate need for hand-control as is found with the equatorial, owing to inequalities of driving and the effect of refraction. In this case it is necessary to watch not one, but two stars in order to control the orientation of the field. It will probably be found that two eyepieces and two observers are required. Hence the advantage of using a cœlostat—which is that limiting form of siderostat which gives a field of constant orientation—is very evident. Moreover the uniform rotation of the mirror about a fixed axis requires the simplest possible mechanism. But on the other hand a fixed telescope in conjunction with a cœlostat commands only one definite declination in the sky, and other declinations must be reached by moving

the telescope either actually or virtually, the latter being the case when the rays reflected by the cœlostat are deflected to the fixed telescope by a subsidiary plane mirror. The Snow telescope of the Yerkes Observatory is an example of the use of a second mirror in this way. The conditions on which the efficient working of the combination depends deserve some consideration, for no investigation of them seems to have been published. Before examining this question, however, it is intended to discuss the possibilities presented by a single mirror which is mounted in a simple way—namely, such as to allow independent rotations about two orthogonal axes.

3. Let us suppose in the first place that the mirror is equatorially mounted, the normal to the mirror corresponding to the axis of the telescope; so that the normal can be adjusted to any desired N.P.D. and driven in right ascension by clockwork. The instrument becomes an ordinary cœlostat when the N.P.D. is 90°, and the velocity in R.A., N, is half the actual

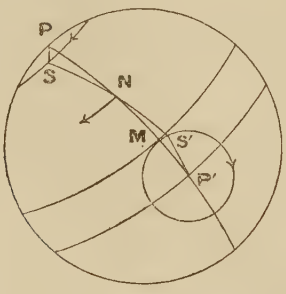


FIG. 1.

angular velocity of the sky, n. By adjusting the declination suitably any desired object can be reflected into the field of the telescope, and the combination can be used for visual observations or for photography when a practically instantaneous exposure is sufficient.

When the normal is inclined to the polar axis at a constant angle ν, and the mirror is driven at a uniform angular velocity N, it is easy to find a simple geometrical representation of the motion of the field. Let it be supposed for the moment that the opposite velocity, $-N$, is impressed on the mirror and the stars. The mirror is thus brought to rest, while a star S (fig. 1) describes its circle round the pole P with the angular velocity $n-N$. Let the ray from P be reflected in the direction P′ and the ray from S in the direction S′. The spherical triangles SNP, S′NP′ are altogether equal and consequently S′ describes round P′ a circle with the same radius and the same velocity $n-N$ as S describes round P.

Now let the angular velocity N be restored to the stars and

the mirror. Then P′ rotates uniformly round P at a distance 2ν with the angular velocity N, carrying with it a picture of the sky which rotates uniformly with respect to PP′ with the velocity $n-N$.

If $PS=P'S'=\Delta$, the circle whose centre is P′ and radius Δ touches the circle whose centre is P and radius $2\nu-\Delta$. Let M be the point of contact. Owing to the rotation of M round P′, the linear velocity of M on the sphere has a component proportional to $(n-N)\sin\Delta$ in one direction, and owing to the rotation of PP′ about P, it has a component proportional to $N\sin(2\nu-\Delta)$ in the opposite direction. Hence if

$$(n-N)\sin\Delta = N\sin(2\nu-\Delta)$$

or

$$\frac{N}{n} = \frac{\sin\Delta}{2\sin\nu\cos(\nu-\Delta)} \quad \ldots \quad \ldots \quad (1)$$

the point M is instantaneously at rest. It follows that the motion of the field may be represented as due to the rolling of the spherical cap whose centre is P′ and radius Δ on the rim of the spherical cap whose centre is P and radius $2\nu-\Delta$. The reflected ray from every star therefore traces a spherical roulette. The motion can also be ascribed to the rolling of one right circular cone on another whose axis is the polar axis.

4. In the case of the properly adjusted cœlostat $\nu = 90°$ and $n = 2N$, so that equation (1) is satisfied identically. The moving circle rolls on itself and the field is permanently stationary, so that a telescope pointed at the mirror will always see the same stars. If the rate of driving is $\frac{1}{2}n$ as for the cœlostat, but the N.P.D. of the normal to the mirror, ν, is not 90°, the rolling and fixed circles are equal but not coincident, and the radius of each is ν. If (ρ, θ) are the polar coordinates of S′, the sides of the triangle S′PP′ are ρ, 2ν, and Δ; and if the angle at P′ is χ, the angle at P is $\chi-\theta$. Hence by eliminating the angle at S′ from two of Delambre's analogies which involve the difference of the angles at P and P′, the polar equation of the locus of S′ on the sphere is easily found to be

$$\sin^2\rho+\sin^2\Delta-2\sin\Delta\sin\rho\cos\theta = \cot^2\nu\,(\cos\rho-\cos\Delta)^2$$

The orthogonal projection of this curve on the equatorial plane can be found in polar coordinates by putting

$$r\cos\phi = \sin\rho\cos\theta-\sin\Delta,\ r\sin\phi = \sin\rho\sin\theta$$

The result of making this substitution is to obtain the ordinary equation of a *limaçon* referred to its pole, which is clearly the projection of the point obtained when $\nu = \frac{1}{2}\pi$, *i.e.* with a cœlostat in perfect adjustment. This theorem is due to Professor Turner (*Monthly Notices*, lvi. p. 419).

5. Since in the general case the motion of the sky as viewed

in the rotating mirror is that which arises from the rolling of one circle on another, it appears that the mirror introduces a complexity which is absent in the simple rotation of the sky as viewed directly. But the fact that a star on the instrumental meridian can be reflected in a chosen direction and by a suitable rate of driving kept stationary even for an instant suggests that it is not without interest to examine the state of things when the star is followed to some distance from the meridian. By a proper adjustment of the rate of driving, provided the clock is capable of a considerable range in this respect, the reflexion of any star can be maintained on the meridian, and its motion in this plane may be compensated by moving the plate-holder. The effect is comparable with that which would be caused by an exaggerated refraction. There is also a rotation of the field, but this would equally be the case if a siderostat were used. Consequently if the motion in declination of the reflected ray could be shown not to exceed a manageable amount, the method would appear to be practicable, and the extremely simple mounting of the mirror gives the question some practical importance.

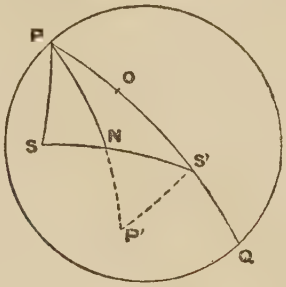


FIG. 2.

6. Let PQ (fig. 2) represent the instrumental meridian, N the direction of the normal to the mirror, S′ the reflexion of the star S, and P′ the reflexion of the pole P, so that $PP' = 2\nu$ and $P'S' = PS = \Delta$. Let $PS' = \rho$, $QPP' = H$, and $QPS = h$; hence $PP'S' = SPP' = h - H$. The relations which are required are arrived at by considering the triangle S′PP′. The first is the relation between H and h necessary in order that S′ may remain on the fixed meridian. This is

$$\cot \Delta \sin 2\nu = \cos 2\nu \cos (h-H) + \sin (h-H) \cot H$$

which can be put in a form to deduce h from H thus:

$$\begin{aligned} \cot \Delta \sin 2\nu \sin H &= \sin h \,(\cos 2\nu \sin^2 H + \cos^2 H) \\ &\quad - \cos h \sin^2 \nu \sin 2H \\ &= \sin (h-K) \,.\, (1 - \sin^2 H \sin^2 2\nu)^{\frac{1}{2}} \\ \text{where} \quad \sin K &= \sin 2H \sin^2 \nu \,.\, (1 - \sin^2 H \sin^2 2\nu)^{-\frac{1}{2}} \end{aligned} \Bigg\} \ldots \quad (2)$$

The corresponding clock-rate is obtained by differentiating the first equation between h and H. This gives

$$\frac{N}{n} = \frac{dH}{dh} = \frac{2 \sin^2 \nu \sin^2 H \sin (h-H) + \sin H \cos h}{2 \sin^2 \nu \sin^2 H \sin (h-H) + \cos H \sin h}$$

or

$$(n+N)/(n-N) = 4 \sin^2 \nu \sin^2 H + \sin (h+H)/\sin (h-H) \quad \ldots \quad (3)$$

The displacement of the point S′ on the meridian, reckoned from the position corresponding to S on the meridian, is $\rho + \Delta - 2\nu$. Now one of Napier's analogies gives

$$\tan \tfrac{1}{2}(\rho+\Delta) = \cos \tfrac{1}{2}(h-2H) \sec \tfrac{1}{2}h \tan \nu \quad \ldots \quad (4)$$

which leads to

$$\sin \tfrac{1}{2}(\rho+\Delta-2\nu)/\sin \tfrac{1}{2}(\rho+\Delta+2\nu) = \tan \tfrac{1}{2}H \tan \tfrac{1}{2}(h-H) \quad \ldots \quad (5)$$

This shows that the displacement may be very small. The angle through which the field has turned since S passed the meridian is QS′P′, which may be denoted by θ. Then clearly

$$\sin \theta = \sin 2\nu \sin H/\sin \Delta \quad \ldots \quad \ldots \quad (6)$$

and

$$\cos \theta \cdot \frac{d\theta}{dh} = \frac{\sin 2\nu \cos H}{\sin \Delta} \cdot \frac{N}{n} \quad \ldots \quad \ldots \quad (7)$$

The formulæ of this paragraph are all exact and suffice to determine with accuracy the relations between the clock-rate and the diurnal motion of the stars, the displacement of the reflected image in the plane of the instrumental meridian and the variable rotation of the field.

7. For our present purpose, however, which is simply to inquire whether the instrument is practicable for comparatively short exposures near the meridian, approximate formulæ are sufficient. The form of the equations shows that the neglect of powers of h higher than the first will not materially affect the results to be expected. Thus from (2)

$$K = 2H \sin^2 \nu$$
$$h - K = H \cot \Delta \sin 2\nu$$

whence

$$H/h = \sin \Delta / 2 \sin \nu \cos (\nu - \Delta) \quad \ldots \quad \ldots \quad (8)$$

By (3) this is also the value of N/n, a result already found in equation (1). The displacement in the plane of the meridian is given by (5) in the form

$$\rho + \Delta - 2\nu = \tfrac{1}{2}H(h-H) \sin \tfrac{1}{2}(\rho+\Delta+2\nu)$$

which becomes, when ρ is given its initial value on the right-hand side,

$$\rho + \Delta - 2\nu = h^2 \sin \Delta \sin (2\nu - \Delta)/4 \tan \nu \cos^2(\nu - \Delta) \quad \ldots (9)$$

The rate of rotation of the field, as given by (7), is

$$\frac{d\theta}{dh} = \frac{\cos \nu}{\cos (\nu - \Delta)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

This rotation is relative to the fixed meridian PQ, while that found in § 3 is relative to the moving meridian PN, and has a different value, namely, $n - N$.

8. Some numerical results can now be given. In the case chosen for the purpose of illustration the latitude is 50° and the telescope is placed in a horizontal position in the local meridian. Hence $\rho = 130°$ initially. The column headed R_1 gives the driving-rate of the mirror in R.A. compared with the corresponding clock-rate of an equatorial. The column headed R_2 contains the initial rotation of the field, expressed as a fraction of one complete rotation in a day. Under the heading D is given the displacement of the image of the guiding star in the first minute after passing the meridian.

Δ	ν	R_1	R_2	D
10°	70°	0·1848	+0·6840	+0″·1901
20	75	·3087	·4512	·2095
30	80	·3949	·2701	·1605
40	85	·4563	+0·1233	+0·0846
50	90	·5000	0·0000	0·0000
60	95	·5306	−0·1064	−0·0849
70	100	·5509	·2005	·1662
80	105	·5625	·2856	·2416
90	110	·5662	·3640	·3100
100	115	·5625	·4375	·3702
110	120	·5509	·5077	·4207
120	125	0·5306	−0·5758	−0·4596

The case of $\Delta = 50°$, in which the N.P.D. of the star is equal to the latitude of the place, satisfies the cœlostat condition. This case marks a change of sign in R_2 and D. The meaning of a positive sign attached to R_2 is that the rotation of the field is clockwise when viewed towards the mirror, the meaning of a negative sign that the rotation is anti-clockwise. Similarly a positive sign attached to D indicates that the displacement after the guiding star has passed the meridian is in the direction of the S. pole, a negative sign that it is towards the N. pole. Within a small distance of the meridian, the total shift of the guiding star

is obtained by multiplying the numbers in the last column by the square of the number of minutes in the hour-angle. The inferences to be drawn from the table are clearly uncertain in the absence of actual experiment. But it would appear at least feasible, with an instrument of the kind considered, to obtain photographs with moderate exposures (up to 20 minutes say) of a belt of the sky extending perhaps 30° south of the zenith.

9. Very steady driving may be expected from a mirror with simple equatorial mounting, and it would probably be best to adjust the clock-rate at the beginning of the exposure of the plate and to apply the adjustment afterwards necessary to the plate-carrier by means of two rectangular slides. By moving the normal to the mirror in N.P.D. the siderostat condition might be partially or completely satisfied, and it would be necessary to apply merely a rotation to the plate. This would complicate the mounting of the mirror, and would probably interfere with the steadiness of the driving. Yet it is a matter of interest to examine the nature of the motions which must be communicated to a mirror capable of rotating about two orthogonal axes in order that it may satisfy the siderostat condition. This problem differs from the one previously considered merely in the fact that ρ is constant and ν variable, instead of the converse. The triangle PS′P′ (fig. 2) has for its angles H, $\pi-\theta$ and $h-$H, and for its sides Δ, 2ν and ρ. Hence three of Napier's analogies give immediately

$$\tan \tfrac{1}{2}\theta = \frac{\cos \tfrac{1}{2}(\rho+\Delta)}{\cos \tfrac{1}{2}(\rho-\Delta)} \tan \tfrac{1}{2}h \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$\tan \nu \cos \tfrac{1}{2}(h-2\mathrm{H}) = \cos \tfrac{1}{2}h \tan \tfrac{1}{2}(\rho+\Delta)$$

$$\tan \nu \sin \tfrac{1}{2}(h-2\mathrm{H}) = \sin \tfrac{1}{2}h \tan \tfrac{1}{2}(\rho-\Delta)$$

The first of these gives the law of rotation of the field, discussed by Cornu. The other two equations lead to

$$\left.\begin{aligned} \tan \nu \sin \mathrm{H} &= \frac{\sin \Delta \sin h}{\cos \rho + \cos \Delta} \\ \tan \nu \cos \mathrm{H} &= \frac{\sin \rho + \sin \Delta \cos h}{\cos \rho + \cos \Delta} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

or

$$\left.\begin{aligned} \tan \nu &= \frac{(\sin^2 \rho + \sin^2 \Delta + 2 \sin \rho \sin \Delta \cos h)^{\frac{1}{2}}}{\cos \rho + \cos \Delta} \\ \tan \mathrm{H} &= \frac{\sin \Delta \sin h}{\sin \rho + \sin \Delta \cos h} \end{aligned}\right\} \quad \ldots \quad (13)$$

which express the laws of motion about the two axes.

10. From these may be deduced the character of the motion about two axes, of which one is fixed in the instrumental meridian plane and inclined to the polar axis at an angle α. Let O represent (fig. 2) the direction of the fixed axis, and let

$ON = \nu'$ and $QON = H'$. The form of equations (12), which give the "standard coordinates" of the direction of the normal to the mirror, suggests a transformation in rectangular coordinates. In the polar system

$$x = \sin\Delta\sin h, \quad y = \sin\rho + \sin\Delta\cos h, \quad z = \cos\rho + \cos\Delta$$

If the axes are turned through an angle a about the axis of x,

$$x' = x, \quad y' = y\cos a - z\sin a, \quad z' = y\sin a + z\cos a$$

In the new system, as in the old,

$$\tan\nu'\sin H' = x'/z', \quad \tan\nu'\cos H' = y'/z'$$

Hence the motion of the mirror, with respect to the new axes, is given by

$$\left.\begin{aligned}\tan\nu'\sin H' &= \frac{\sin\Delta\sin h}{\cos(\rho-a)+\cos a\cos\Delta+\sin a\sin\Delta\cos h}\\ \tan\nu'\cos H' &= \frac{\sin(\rho-a)-\sin a\cos\Delta+\cos a\sin\Delta\cos h}{\cos(\rho-a)+\cos a\cos\Delta+\sin a\sin\Delta\cos h}\end{aligned}\right\} \quad (14)$$

11. From these results a number of particular cases can be deduced by assigning different values to a. If, for example, the instrumental is coincident with the local meridian, and a is the co-latitude, the equations (14) characterise the motion when the mirror has an altazimuth mounting. But no cases beyond those already known appear to present simple types of motion which can be derived from clockwork without complicated mechanism. Hence, unless such motions can be obtained by electric motor-driving under automatic control, the problem of the siderostat is apparently not facilitated by resolving the motion of the mirror about two orthogonal axes. This is true even if a is allowed to be a function of Δ, *i.e.* if the main axis, instead of being fixed, is adjusted according to the declination required—an arrangement only practicable with a small mirror. One case of some interest is that in which the fixed axis is directed towards the centre of curvature of the sphero-conic traced by the normal to the mirror, corresponding to the point where the meridian is crossed. The expressions occurring in equations (14) may be expanded in powers of $T = \tan\frac{1}{2}h$, thus

$$\tan\nu'\sin H' = A_1T + A_3T^3 + \ldots, \quad \tan\nu'\cos H' = B_0 + B_2T^2 + \ldots$$

so that

$$\tan^2\nu' = B_0^2 + (A_1^2 + 2B_0B_2)T^2 + \ldots$$

The coefficient of T^2 involves a, and can be made to vanish by a suitable choice of this angle. The result of a little reduction is to give the value

$$a = \tfrac{1}{2}\rho - \tan^{-1}(\tan\tfrac{1}{2}\Delta\tan^2\tfrac{1}{2}\rho) \ldots \quad \ldots \quad \ldots \quad (15)$$

It follows that when the fixed axis has the position indicated by this equation, the variation of ν' depends on the fourth power of the time, and is at first very slow. When the example of § 8, in which $\rho = 130°$, is examined, equation (15) gives the following numerical results :

Δ	α	Δ	α	Δ	α
10°	43° 5′	50°	0° 0′	90°	12° 44′ S.P.
20	25 58	60	4 22 S.P.	100	14 40 „
30	14 4	70	7 45 „	110	16 21 „
40	5 51	80	10 28 „	120	17 51 „

12. The advantage of the form of mounting which has been examined lies in the stability which it renders possible, a feature in regard to which the ordinary form of siderostat is liable to grave suspicion. It is not impossible that an entirely satisfactory method may be available for producing the motions defined by equations (13). It is easy to devise a linkage which contains the solution of the problem, and though link-motions are not free from objection, any method which is based on a new principle, as this appears to be, possesses a certain interest. Let O and C (fig. 3)

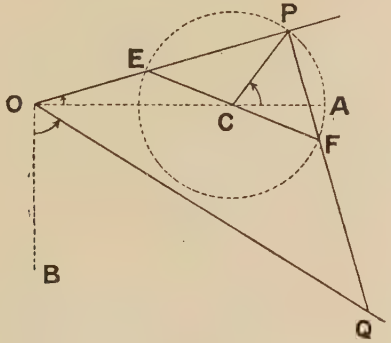


Fig. 3.

be two points fixed at a distance apart equal to sin ρ. Let CP, CE and CF be each equal to sin Δ on the same scale and PQ to cos ρ + cos Δ. The lines OP, PQ, and OQ represent slotted bars, so that the points P and E are free to move along OP, F along PQ, and Q along OQ. Then if CP rotates uniformly in twenty-four hours so that the angle ACP is h, A lying on OC produced, we have

$$\tan \mathrm{AOP} = \sin \Delta \sin h / (\sin \rho + \sin \Delta \cos h)$$

$$\tan \mathrm{OQP} = \mathrm{OP}/\mathrm{PQ}$$

$$= (\sin^2 \rho + \sin^2 \Delta + 2 \sin \rho \sin \Delta \cos h)^{\frac{1}{2}} / (\cos \rho + \cos \Delta)$$

for OPQ is a right angle. Hence AOP is the angle H and OQP is the angle ν. If OB is perpendicular to OA, the angle BOQ is

equal to $\nu+H$. It follows that if the point O is on the polar axis and the plane of the linkage is perpendicular to this axis, a wheel of which OP is a radius rigidly connected with the mirror will communicate the required motion in R.A., while a second wheel of which OQ is a radius, if suitably geared round the frame to a wheel on the moving axis, will communicate the required motion in N.P.D. This constitutes a solution of the siderostat problem. The adjustment for different declinations involves the ratios of OC, CP and PQ. Probably the most convenient method would be to keep CP constant and to vary OC and PQ.

13. The use of a cœlostat in conjunction with a second mirror, which deflects the rays from the cœlostat into a permanently fixed telescope, has been mentioned in § 2, and the possibilities of this arrangement may now be considered. The position of the telescope is supposed to be horizontal, and can be specified by the

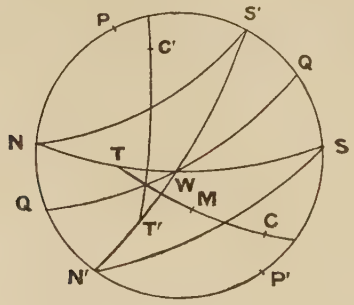


Fig. 4.

azimuth to which it is directed, and the azimuth and Z.D. of the perpendicular drawn from the centre of the cœlostat to the axis of the telescope. If the latter coordinates are A and Z respectively, the first-mentioned azimuth is $A\pm\frac{1}{2}\pi$, where the distinction between the two signs at our disposal corresponds to a complete reversal of the telescope. It is clear that the mirror (by which is meant here the subsidiary plane mirror) must move along the axis of the telescope. This axis, together with the centre of the cœlostat, determines a plane, or rather a half-plane, in which lie all the rays which can be brought into the centre of the field of the telescope. To this half-plane corresponds on the sphere (fig. 4) a great semicircle, with its extremities T, R on the horizon NWS, inclined to this circle at an angle $Z-\frac{1}{2}\pi$, and having for its middle point the point C, which represents the perpendicular from the centre of the cœlostat to the axis of the telescope. If a point on the semicircle has the N.P.D. Δ' and the hour-angle h', while the N.P.D. and hour-angle of a star

reflected in the direction of the point are Δ and h, and if the hour-angle of the normal to the cœlostat is H, then

$$\Delta' = \pi - \Delta, \qquad h' = 2\mathrm{H} - h$$

Each point on the axis of the telescope corresponds to a definite declination, and the effect of turning the cœlostat in any manner is to bring different hour-angles into view.

The relations between the position of any point on the sphere and of its reflexion in the cœlostat are equivalent to two steps which it is convenient to consider as made successively. These consist in passing, first, to the image of the point in the plane of the equator; and, secondly, in passing to the image of this image in the plane of the meridian which contains the normal to the cœlostat. Now let N′WS′ be the image of the horizon in the equatorial plane, and let T′C′R′ be the image of the semicircle TCR. Then clearly T′C′R′ is also a semicircle with its extremities on N′WS′, and inclined to N′WS′ at the same angle as TCR to NWS. It lies wholly on one side of the image of the horizon, and its position decides the range of declination which the telescope can command. Then by suitably choosing the position of the normal to the cœlostat any point in T′C′R′ can be brought, subject to certain limitations which must be examined, to any desired hour-angle in the sky.

14. The limitations referred to are imposed by the maximum angle of incidence on the cœlostat which can in practice be allowed. This angle cannot exceed 60°, in which case the effective aperture of the cœlostat is reduced by one half, and ought, if possible, to be less than 45°. Denoting the angle of incidence by i and its limiting value by I, we have

$$\cos i = \sin\Delta \cos \tfrac{1}{2}(h' - h) \not< \cos \mathrm{I}$$

and so we obtain for the two values of I the following limiting values of $h' \sim h$:

I = 60°.

Δ =	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°
$h' \sim h$ =	0h·0	5h·2	6h·6	7h·3	7h·7	7h·9	8h·0	7h·9	7h·7	7h·3

I = 45°.

Δ =	45°	50°	60°	70°	80°	90°	100°	110°	120°
$h' \sim h$ =	0h·0	3h·0	4h·7	5h·5	5h·9	6h·0	5h·9	5h·5	4h·7

By the nature of the case the sky in the neighbourhood of the pole to a distance equal to $\frac{1}{2}\pi - \mathrm{I}$ cannot be brought within the field of the telescope.

15. The circumstances of the reflexion at the surface of the subsidiary mirror need explanation. The point O (fig. 5) is the centre of the cœlostat and the centre of the sphere represented in fig. 4. The axis of the telescope, which is pointed in a direc-

tion parallel to OT, is CM, C being the foot of the perpendicular from O and M the position of the mirror. Let $OC = d$, $MC = x$, and $TOM = a$, so that

$$x = d \cot a$$

The normal to the mirror MV is in the plane TOC, and the angle VMC is $\frac{1}{2}a$. The axis about which the mirror must be capable of turning is normal to the plane TOC and so is fixed in direction. The angle of incidence on the mirror being $\frac{1}{2}a$, we have $a \geqslant 2I'$, where I' is the greatest angle of incidence allowed. The ray for which $a = 0$ cannot be brought into the field of the telescope ; and if an upper limit is assigned to x, the distance of the mirror from C, there is a corresponding lower limit to a.

To complete the investigation of the circumstances on which the adjustment of the mirror depends, it is necessary to find the relations between a, Δ' and h' in terms of the data which express

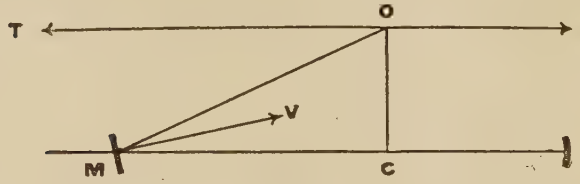


Fig. 5.

in a natural form the position of the telescope relative to the cœlostat. These are (fig. 4) $NT = B$, Z the Z.D. of C, and $PN = \phi$, the latitude. It is convenient also to introduce the dependent constants p and h_0, the N.P.D. and hour-angle of T, and τ the angle PTN, so that $PTM = \frac{1}{2}\pi + Z - \tau$. Then we obtain

$$\left.\begin{aligned} \cos \Delta' &= \cos p \cos a - \sin p \sin a \sin (Z - \tau) \\ \sin p \cot a &= -\cos p \sin (Z - \tau) + \cos (Z - \tau) \cot (h_0 - h') \end{aligned}\right\} \qquad (16)$$

where

$$\begin{aligned} \cos p &= \cos B \cos \phi \\ \cot h_0 &= -\cot B \sin \phi \\ \cot \tau &= \sin B \cot \phi \end{aligned}$$

With these, or equivalent, formulæ it is possible to tabulate in any given case the corresponding values of a, Δ' and h', and with the tables so formed to adjust the position and inclination of the mirror according to the N.P.D. required, and to make the corresponding setting of the cœlostat.

The formulæ connecting Z, A, the Z.D. and azimuth (reckoned

from N through W) of the point C with Δ_o', h_o', the N.P.D. and hour-angle of the same point, may also be useful. They are

$$\left.\begin{aligned} \cos Z &= \cos \Delta_o' \sin \phi + \sin \Delta_o' \cos \phi \cos h_o' \\ \cot A \sin h_o' &= \cot \Delta_o' \cos \phi - \sin \phi \cos h_o' \end{aligned}\right\} \quad \dots \quad (17)$$

16. The principles discussed above enable us to examine the nature and relative advantages of different arrangements which are possible. In choosing a suitable disposition of the instruments there are three main factors to be considered: namely, (*a*) the effective range of declination within reach; (*b*) the hour-angle of the part of the sky under observation; and (*c*) the angle of incidence at the cœlostat. The two latter are, as we have seen, closely related, while the first factor depends entirely on the position of the arc T′C′ (fig. 4) in the sky. A few typical arrangements will now be briefly examined.

(1) T′ coincides with S′, and C′ is taken on S′S. From the corresponding position of TC it is clear that this means that the telescope is pointed due south, and its axis passes directly *over* the cœlostat, for C coincides with the zenith. The whole of the southern meridian is observed under the best possible conditions to a distance from the pole equal to the latitude, the limits of N.P.D. being ϕ and $\pi - \phi$. Except in very high latitudes this position is always suitable for solar work, and is in fact the one adopted for the Snow telescope.* Its maximum efficiency requires a moderately low latitude. In latitude 40°, for example, the meridian can be observed up to and beyond the zenith, and the only part of the sky which lies out of reach is the vicinity of the pole, for which the cœlostat is by its nature inadequate.

(2) T′ coincides with S′ and C′ is taken on S′P. The telescope is pointed due south and its axis passes *underneath* the cœlostat. This position supplements the former and brings under observation that part of the meridian whose limits of N.P.D. are ϕ and $\frac{1}{2}\pi - I$.

(3) Still keeping the telescope pointed south, we may place its axis on a level with and west of the cœlostat, so that C′ comes to W. For the arc S′W the limits of N.P.D. are ϕ and $\frac{1}{2}\pi$. Without making the angle of incidence at the cœlostat excessive, the available part of the sky can be observed on or near the meridian. But this case is clearly inferior to (1), and it is fairly evident that with the telescope pointed south it is impossible to place the axis so as to obtain any advantage by combining parts of the ranges in N.P.D. covered by cases (1) and (2).

(4) T′ coincides with N′ and C′ lies on N′P. The telescope is pointed due north, and its axis passes *underneath* the cœlostat. This case at first sight offers the greatest advantage with regard to the range in N.P.D., for the arc extends over N′P and corresponds to the whole of the visible sky. But if we apply the

* This is only approximately true as the telescope is at present installed on Mount Wilson. See *Astrophysical Journal*, 1905 March, p. 163.

results found in § 14 we shall find that the effective range is greatly curtailed; and even so it is necessary to have a large angle of incidence at the cœlostat, and to work at a quite impracticable hour-angle. This case presents no advantage.

(5) The telescope is pointed due west. This position may be recommended by local considerations as to available space, and these must be taken into account as well as the factors already mentioned. In this case T′ coincides with W and C′ lies on the meridian and must be placed either on QP or on QS, so that the range of sky is entirely above or below the equator. (*a*) If below, S is the best position for C′; and if this is adopted, C coincides with S′, and the perpendicular from the cœlostat to the axis of the mirror is inclined southward at an angle $2\phi-\frac{1}{2}\pi$ to the upward vertical, the range in N.P.D. being from $\frac{1}{2}\pi$ to $\pi-\phi$. (*b*) If, as would naturally be the case in a moderately high latitude, north declinations are required, C′ may be taken at a distance $\frac{1}{2}\pi-\mathrm{I}$ from P, so as to obtain the greatest range of declination possible at the most favourable hour-angle. The perpendicular from the cœlostat to the axis of the mirror is inclined southward at an angle $\phi+\mathrm{I}$ to the upward vertical. The range covered in N.P.D. extends from $\frac{1}{2}\pi-\mathrm{I}$ to $\frac{1}{2}\pi$. This case is therefore superior to (3) at places where the co-latitude is less than the greatest angle of incidence at the cœlostat which is considered allowable.

17. Enough has been said with regard to the use of a cœlostat in conjunction with a fixed horizontal telescope and a second mirror. Positions of the telescope other than horizontal will not be discussed here. But mention may be made of the case in which the telescope is directed to the zenith. Here the second mirror must be moved vertically; and if the cœlostat is placed to the north of the axis of the telescope, a considerable part of the southern meridian, interrupted at the equator itself, can be brought within the field, ranging, in latitude 45°, practically from the horizon to the zenith. The idea of a concave mirror placed a hundred feet or so beneath the surface of the earth is a strange one; but the plan might possibly avoid the defects of the horizontal position: namely, the bad definition due to the unsteadiness of the atmosphere near the ground and the variability in the focal length owing to changes in the temperature of the mirror.

18. In addition to the cœlostat we have as a particular case of the siderostat the polar heliostat of Fahrenheit, which may be regarded as the prototype of the modern coudé form of equatorial. The essential difference between the combination of a telescope and a polar heliostat and the Sheepshanks telescope at Cambridge, for example, consists merely in the order of the mirror and the object-glass. The inconvenience of the instrument arises from the rotation of the field which is obviated by the cœlostat. It is also necessary to place the axis of the telescope parallel to the polar axis. This position can, however, be

avoided by the use of a second mirror fixed so as to deflect the rays from the heliostat in any desired direction. In this way a polar heliostat can be used in conjunction with either a horizontal or a vertical telescope.

19. The contents of this paper may be summarised thus :

(1) §§ 1 and 2 are introductory.

(2) §§ 3 and 4 deal with the geometry of a mirror caused to rotate with uniform angular velocity about the polar axis, which is inclined at a constant angle to the plane of the mirror.

(3) §§ 5–8 contain a discussion of the properties of a mirror similarly mounted, but rotated with such a (variable) velocity that the image of a particular star is maintained on the instrumental meridian.

(4) §§ 9–11 deal with the motion of a siderostat considered as a mirror capable of rotation about any two orthogonal axes, one of which is fixed in the plane of the instrumental meridian.

(5) § 12 describes a linkage which is capable of producing the motions required by a siderostat which is mounted equatorially.

(6) §§ 13–15 deal with the geometrical conditions on which the efficient use of a second mirror in conjunction with a cœlostat depends when the telescope is fixed in a horizontal position, and with the necessary adjustment in the position of the mirror.

(7) § 16 is devoted to an examination of certain typical positions of the telescope.

(8) §§ 17 and 18 refer to the use of a cœlostat with a telescope pointed to the zenith and to the possible advantage of a polar heliostat combined with a fixed mirror.

University Observatory, Oxford :
1905 *March* 8.

The Optical Sine-condition. By A. E. Conrady.

The remarkable theorem which forms the subject of this paper was brought to the notice of astronomers in general by the stipulations agreed upon at the Paris Congress for standardising the construction of the astrographic telescopes ; for, on the recommendation of Dr. Steinheil, one of the German delegates, it was laid down that the objectives must fulfil the sine-condition, so as to insure the formation of symmetrical images in the outer part of the relatively large field that had to be covered.

A short history of this theorem and a simple proof of it may therefore be acceptable.

Owing to its close association with the complicated defect known to opticians as " coma," it is a matter of course that this

condition is found approximately fulfilled by all reasonably successful lens-systems, even those made by purely cut-and-try methods.

But in the case of the famous object glasses made by Fraunhofer in the early part of the last century, more particularly in the much discussed objective of the Königsberg heliometer, the fulfilment of the condition is such an exceedingly close one as to almost force one to assume that it was arrived at deliberately by computation. But the latter was most probably an elaborate trigonometrical one for an oblique pencil of rays, which, thanks to the sine-condition, modern computers can generally do without.

The first theoretical paper which gave something nearly akin to the modern form of the sine-condition was a classical one by v. Seidel,* in which the elementary dioptric theory was extended so as to include all terms of the third order, the result being that v. Seidel showed that a lens-system must fulfil five separate conditions in order to give a sharp and plane image of an extended object ; the second of these five conditions corresponds to the modern sine-condition, but it is necessarily in an approximate algebraical form, as all trigonometrical functions were replaced by their development in series, and only the first two terms retained.

Eight years later, in 1863, R. Clausius,† in one of the numerous investigations which he had to undertake in defence of his second law of thermodynamics, was led to a very careful study of the concentration of radiant heat, and arrived, *inter alia*, at a differential equation which embodies the sine-condition, and the value of which for optical purposes he pointed out. To Clausius, therefore, belongs the honour of first discovery, although this has only been recognised in comparatively recent times.

The two names which are usually associated with the discovery are those of Abbe and v. Helmholtz. The former‡ alludes to the sine-condition in one sentence of a paper published in 1873 ; but that has secured him priority over v. Helmholtz, as the latter himself acknowledged in a P.S. to his own paper§ of the same year, in which the theorem was for the first time clearly stated as a separate optical one, and proved in a manner somewhat resembling that of Clausius.

Since then a very popular though not quite rigorous proof of the sine-condition, which has found its way into many books, has been supplied by Hockin.‖

The existing proofs of this important theorem are either rather involved or else wanting in validity, inasmuch as the proof only extends to rays proceeding in a plane containing the

* *Ast. Nach.* 1027–1029 (1855).

† R. Clausius, *Pogg. Ann.* cxxi. p. 1 (1864).

‡ Abbe, M. Schultze's *Archiv für mikroskopische Anatomie*, ix. pp. 413–468 (1873).

§ Helmholtz, *Pogg. Ann.*, Jubelband, p. 557 (1874).

‖ Hockin, *Journal Roy. Microscopical Soc.*, Ser. II., vol. iv. p. 337 (1884).

optical axis; most of them also fail to bring out clearly the result of non-fulfilment, and none deal with the significance of the theorem in the presence of spherical aberration.

The following proof, simple as it is, seems to be free from these objections.

In fig. 1 let OZ be the optical axis of a system of lenses or mirrors, or both combined, no other restriction being imposed than that all the refracting or reflecting surfaces shall be continuous surfaces of rotation with OZ as axis, and the system therefore a "centred" one. The rays from O shall be allowed to enter the system through a narrow zone marked by the circle A with C (on OZ) as centre; after passing through the system they are assumed to emerge by the zone defined by the circle A′ with C′ as centre, and to come to a focus at O′. That all rays of such a zone must come to a common focus follows from the symmetry of the entire system round the axis OZ. For the same reason any one ray, such as O—A—A′—O′ must always remain in that plane containing the axis OZ in which it started from O; the angles β_1 and β_1' formed by the incident and emergent rays with the optical axis must be the same for all rays of the zone under consideration, and the paths of all such rays must be equal, both as to their entire length and as to corresponding constituent parts.

Owing to these properties of a centred optical system it is possible to discuss, without separate computation of the paths of rays through the system, the formation of the image of any point P near the optical axis, viz. so close to O that to any ray passing from O through the system we can find a corresponding ray from P which only forms small angles of the first order with it, and which does not become separated from it by more than small quantities of the first order anywhere within the system.

This generalisation depends on Fermat's theorem of the minimum optical path, according to which light passes from one point to another along the path which requires the least time * to traverse. For, according to the definition of a maximum or minimum, it follows at once that any path which agrees with a known optically determined path within small quantities of the *first* order is of the same length within small quantities not exceeding the *second* order, and is therefore sensibly equal to it. We are, therefore, justified in assuming that the paths traversed within the optical system by light from P are sensibly equal to those followed by light from O, and we can base our study of the image of P on a comparison of the paths outside the system.

In order that P′ may be the image of P all optical paths between P and P′ must be of equal length, for that is the

* In certain cases the path assigned to light by the laws of refraction and reflexion is the one which requires the *longest* time. Fermat's theorem, therefore, requires an extension. We must state that light passes from point to point along a path for which the time of transit is *an extreme value*, i.e. either a minimum or a maximum.

condition under which all light from P will arrive at P′ in the same phase of vibration so as to form a bright image.

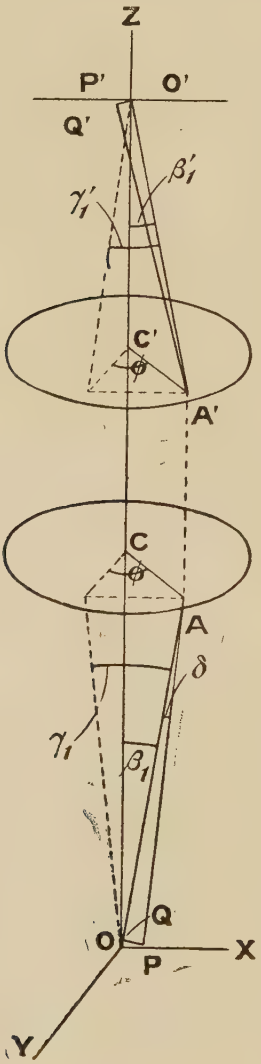


Fig. 1.

We know that all paths between O and O′—viz. the zone under consideration—*are* equal; our task, therefore, reduces itself to comparing any ray from P with its corresponding ray from O,

and by Fermat's theorem we need not consider the portion within the system.

Now OA and PA are two such corresponding rays, and by dropping a perpendicular PQ from P upon PA we see that PA is shorter than OA by OQ. On reference to the rectangular system of coordinates shown in fig. 1, P lying on the axis OX, it is seen that, γ_1 being the angle between OA and the YZ plane, we have

$$OQ = OP \cdot \sin \gamma_1$$

Calling the angle between radius CA and the YZ plane ϕ, we further have

$$\sin \gamma_1 = \sin \phi \sin \beta_1$$

and therefore

$$(1) \quad OQ = OP \cdot \sin \beta_1 \cdot \sin \phi$$

The angle ϕ being the angle between the YZ plane and the plane in which the ray OA proceeds through the system is constant for this ray; hence, by analogy, we can at once write down that the quantity P′Q′ by which A′P′ is longer than A′O′ is given by

$$(2) \quad P'Q' = O'P' \sin \beta_1' \sin \phi$$

(1) and (2) give us the *geometrical* differences between the optical paths OAA′O′ and PAA′P′. But we must compare optical paths, *i.e.* we must bear in mind that the velocity of light is inversely proportional to the refractive index, and that the geometrical paths must, therefore, be multiplied by the index of the medium in which they lie in order to be commensurable. Taking the indices surrounding object and image respectively as n and n', we therefore get the difference of the optical paths O—O′ and P—P′ given by

$$n \cdot \overline{OQ} - n' \cdot \overline{P'Q'} = \sin \phi \{n \cdot OP \cdot \sin \beta_1 - n' \cdot O'P' \sin \beta_1'\}$$

This difference varies, therefore, in general according to the value of ϕ. But it vanishes, and all optical paths between P and P′ become equal when the bracketed term becomes zero; hence a narrow zone of *any* centred optical system will yield sharp images of points not in but near the optical axis, and the position of the image is determined by the condition

$$(3) \quad n \cdot \overline{OP} \cdot \sin \beta_1 = n' \cdot \overline{O'P'} \cdot \sin \beta_1' \text{ *}$$

$\frac{O'P'}{OP}$ is obviously the magnification of the image, for which we

* It is interesting to note the close resemblance of this equation to the so-called optical invariant $OP \cdot n \cdot \tan \beta_1$ which is obtained in geometrical optics on the assumption of strictly collinear relation between object-space and image-space. The difference between the two proves that optical systems of wide aperture do not possess the property of collinearity.

will introduce the symbol M. Introducing this, we arrive at the fundamental equation

$$\text{I.} \quad \mathrm{M} = \frac{n \sin \beta_1}{n' \sin \beta_1'}$$

which thus expresses a universal property of centred optical systems, whether spherically corrected or not, and which in this generality would seem to be new. Although I have not specially insisted upon it, it will be obvious that all the arguments used remain valid, not only for the case graphically shown in fig. 1 of a real inverted image, but also for virtual images, and even though the rays should cross the axis once or several times within the system.

Equation I. at once becomes the sine-condition in its usual form if, assuming a spherically corrected system, we demand that all zones of the system shall concentrate the light from P in the same point ; for that is equivalent to demanding that all zones shall have the same magnifying power. Hence, taking β and β' as the general symbols for conjugate angles of convergence of any ray before and after passing through the system, we have the condition under which a system spherically corrected for the optical axis will also correctly depict points *near* the optical axis.

$$\text{II.} \quad \frac{n \sin \beta}{n' \sin \beta'} = \text{constant} = \text{magnification.}$$

Owing to the comparative uselessness of lenses which, whilst spherically corrected in the optical axis, offend against this condition, Professor Abbe has suggested that the term "aplanatism" should be defined as *including* the fulfilment of this condition, which may well be called the optical computer's best friend ; for on reference to my paper on "Chromatic Correction of Object-glasses" * it will be seen that the sines of the angles β are habitually used to define the position of rays ; hence the sine-condition enables the computer to check the usefulness of the lens he is computing by simply finding the differences of a few pairs of logarithms which he has already used in the computation for the axial point.

But, apart from its practical usefulness, the sine-condition provides a short cut in many other optical problems. The well-known rule, for instance, that the magnification of a telescope is found by dividing the clear diameter of the object-glass by the diameter of the small bright circle seen at the eyepiece-end is a necessary consequence of the sine-condition which leads to this ratio, absolutely regardless of the particular arrangement of mirrors and lenses of which the instrument may be composed.

The way in which I have deduced the condition renders it absolutely clear what must be the result of non-fulfilment : The

* *Monthly Notices*, January 1904.

different zones will have different magnifying powers; they will therefore produce images of an extra-axial point which occupy different positions on a radial line, and which together form a *line* instead of a point. Hence there can be no distinct image except in the immediate vicinity of the optical axis. In telescope-objectives the differences of magnification that are possible with the usual types are comparatively small; hence all of them answer fairly well when used *with eyepieces restricted to the optical axis.* But in microscope-objectives it is possible to produce systems which, whilst perfectly corrected for spherical and chromatic aberration in the optical axis, have differences of magnification of as much as 50 per cent. between centre and margin. Obviously this makes the formation of a recognisable image of an extra-axial point impossible, and we have therefore the remarkable possibility of lens-systems the useful field of which is of the same order as their resolving power, *i.e.* which might separate two points if placed symmetrically to the optical axis, but which would break down if the object left the optical axis *at all.*

The importance of this theorem for photographic telescopes arises from the fact which has been repeatedly proved by very laborious computations by the staff of Dr. Steinheil in Munich, that telescope-objectives which *rigorously* fulfil the sine-condition produce perfectly symmetrical images of stars up to the extreme limits of the usual astrographic field; hence the recommendation of the Paris Congress.

For telescopes the equation expressing the condition allows of a convenient modification. Telescopic objects are at a considerable distance from the instruments, so that β is a very small angle, and their angular extent rather than their linear magnitude is known. Let δ be the angle subtended by OP as seen from A; then we can replace OP in equation (3) by its equivalent OA . δ and write

$$\mathrm{OA} \,.\, \delta \,.\, n \,.\, \sin\beta = \mathrm{O'P'} \,.\, n' \,.\, \sin\beta'$$

n, the index of the medium surrounding the object, may for the telescope be put $= 1$, and OA $\sin\beta$ is evidently the semi-aperture CA which we will call y. Hence we have the sine-condition for the telescope in the form

$$\text{III.} \quad \frac{y}{n' \sin\beta'} = \frac{\mathrm{O'P'}}{\delta} = \text{constant}$$

whence it is evident that with an object at a very great distance $\frac{y}{n' \,.\, \sin\beta'}$ is equal to the equivalent focus of the lens-system; for it is the distance at which the image subtends the same angle as that subtended by the object. Of course in most telescopes n' is also equal to unity, the image being formed in air; but the more general form of III. includes such instruments as the one filled

with water which Airy used in a famous experiment concerning the constant of aberration.

As to the significance of the sine-condition in systems affected with spherical aberration, it may there be used to secure symmetrical images by so proportioning the magnification of the different zones to the distance from the lens-system at which they come to a focus that all the cones have a common axis (see fig. 2). As the longitudinal aberration of lens-systems of moderate aperture—like the departure of successive zones from the sine-condition—is proportional to the second power of the aperture, this end can be completely attained in such cases. This extension of the theorem is important, as it can be used considerably to abridge the amount of computation necessary to arrive at a perfect lens, and also because in some cases spherical correction has to be sacrificed to other conditions.

This paper may fittingly be closed with some remarks which may supply the answer to a question which will occur to many,

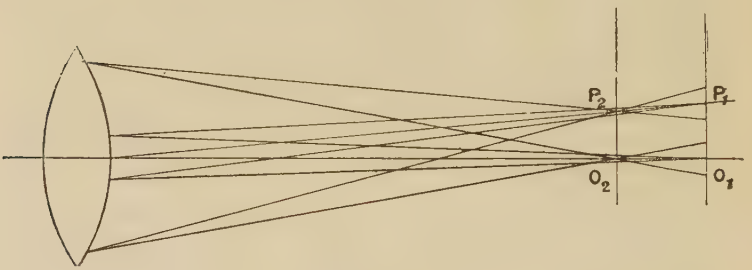


Fig. 2.

whether it is of any use making elaborate computations from which to construct object-glasses when the foremost opticians have claimed and proved that an hour's "figuring" will completely alter the correction of an object-glass. The answer is that this is quite true *so far as spherical correction in the optical axis is concerned*, but that it is wrong when chromatic correction is considered, and hopelessly so when it comes to fulfilling the sine-condition. For the process of "figuring" locally removes layers of glass the thickness of which is of the order of a wave-length of light; in fact, it requires considerable polishing to remove even that thickness; but, if done in the right place, it certainly does completely alter the spherical correction. And inasmuch as it is impossible rigorously to realise the strictly spherical surfaces assumed by the computer, it is indeed useless to carry the computation to great refinement so far as spherical correction is concerned; the result of such misplaced energy almost invariably is a spherically *under*-corrected objective owing to the tendency of the polishing tools slightly to round the extreme edges in the manner which is taken advantage of in

the "figuring" of mirrors. But it is otherwise with chromatic aberration. Sensibly to change the colour for which an objective has minimum focus means a very considerable alteration, one which it would be hopeless to try and effect by *any* polishing process; it means an alteration by turning and subsequent regrinding of the tools employed, and a complete reworking of at least one surface of the objective; or a still greater alteration of two surfaces if a given focus is to be maintained. And if the sine-condition is to be fulfilled, matters become even worse.

The well-known type of object-glass, for instance, which consists of an equi-convex crown and a practically plano-concave flint, offends against the sine-condition to the extent of a difference of magnification between centre and margin, which is of the small order of one-tenth of one per cent. Yet to correct this comparatively slight error and to secure images as symmetrical as should be demanded for photographic purposes, the crown-lens will have to be altered until the curvature of its outside is about half that of the inside, whilst the flint-glass will have assumed a pronounced meniscus-form, if indeed it should be at all possible to carry out so drastic an alteration without making the glasses too thin. In fact, it may be stated, without fear of serious contradiction, that it will always be a hard task to produce by rule of thumb an objective having *minimum focus for a prescribed wave-length*, and that it would be a hopeless enterprise to try to rigorously fulfil the sine-condition without careful computation.

On the Large Sun-spot of 1905 *January* 29–*February* 11, *and Contemporaneous Magnetic Disturbances, observed at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer-Royal.*)

The largest sun-spot as yet photographed at the Royal Observatory, Greenwich, appeared at the east limb of the Sun on 1905 January 28, and passed off at the west limb on February 11. This was the rotation in which it attained its greatest dimensions; but it had been seen in the preceding rotation as well, and reappeared in the third rotation. It was first photographed at Greenwich on January 7, that day being the first occasion on which solar photographs were obtained this year. It is therefore not yet certain as to whether the group formed first in the visible hemisphere or in the one turned from us. If the latter, it would have come into view at the east limb on January 1. The group is now (March 10) completing its third rotation and passing out of sight at the west limb. The Sun was photographed at Greenwich upon five days during the first apparition of the group, upon ten days during the second,

and upon seven days during the third. The photographs taken during the first two apparitions have all been measured and reduced ; but there has been only time enough as yet to measure one taken during the third.

When first seen, on January 7, the group consisted of a scattered stream of rather small faint spots, the leader spot being the only one at all dark or well defined. Many of the faintest of these small spots disappeared on the succeeding days, and on January 12 only four small spots remained.

Date. Greenwich Civil Time.	Projected Area of		Area for Group corrected for Foreshortening		Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian.
	Umbra.	Whole Spot.	Umbra.	Whole Spot.			
1905. d					°	°	°
Jan. 7·540	25	246	13	130	321·5	−18·2	−12·0
11·401	25	167	17	112	323·4	−17·1	+40·7
12·471	12	85	12	77	324·3	−17·0	+55·7
13·473	11	119	16	155	323·7	−17·3	+68·3
14·440	0	7	0	15	320·3	−18·1	+77·7

The projected areas are expressed in millionths of the Sun's apparent disc, the corrected areas in millionths of the Sun's visible hemisphere.

The group on January 12 was 11° of solar longitude in length and about 2° of solar latitude in breadth. Its length on January 7 was 8° of longitude. It began to cross the central meridian January $8^d\ 3^h$, and had completely passed by January $8^d\ 17^h$, the centre of the group passing January $8^d\ 10^h$. The group was not observed on January 15, but must have passed from the visible hemisphere on that day.

The group returned to the east limb on January 28, but no photographs were obtained on that day. It was photographed on January 29, and after that, day by day continuously, except on February 4, 7, and 9, until it reached the west limb on February 11. It was now altogether changed in appearance, being a single spot of enormous dimensions and most complex structure. On February 3 it was 16° of solar longitude in extreme length and 7° of solar latitude in extreme breadth. The principal umbra appeared to be rather small in proportion to the entire area of the spot, and was crossed in several directions by an intricate system of bright bridges ; smaller umbræ, less dark and definite, were scattered irregularly in the widely extended penumbral regions. As the spot neared the west limb the irregular extensions on all sides, but especially in the front and rear, tended to become quite separated from the main spot, which was further divided by two bright bridges, at right angles to its length, extending almost completely across it.

Date. Greenwich Civil Time.	Projected Area of		Area for Group corrected for Foreshortening		Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian.
	Umbra.	Whole Spot.	Umbra.	Whole Spot.			
1905. d					°	°	°
Jan. 29·497	182	1771	327	3190	329·3	−15·7	−75·0
30·499	300	2690	310	2784	329·5	−15·6	−61·6
31·470	420	4222	320	3218	329·2	−15·6	−49·2
Feb. 1·461	641	5119	398	3180	329·8	−15·7	−35·5
2·451	935	6139	508	3339	329·9	−15·2	−22·3
3·458	1037	6293	532	3229	329·3	−15·6	−9·7
5·517	656	5221	348	2771	329·5	−14·8	+17·5
6·672	399	4321	235	2472	328·1	−14·7	+31·4
8·544	201	2664	179	2369	328·4	−14·7	+56·4
10·539	56	440	163	1252	327·2	−14·8	+81·3

The great spot could just be perceived as a very slight dark mark on the west limb on February 11, but it was not possible to measure it.

It was next photographed on February 25, when it had again returned to the east limb. It was now not nearly as large as during the preceding rotation, but was still a very fine group. Its form was now that of an extended stream, the principal spot being very large and complex, with two or three regular spots following it, and a few very small faint spots preceding it. The extreme length of the group on March 4 was 13° of solar longitude, its extreme breadth 5° of solar latitude.

Another great spot, but in the northern hemisphere, appeared at the east limb on March 1. This, like the group of January 28 to February 11, was a single spot of great complexity of detail. It was represented during the preceding rotation by quite a small group, a short stream at the head of a long scattered procession of groups following each other at considerable intervals. The same general region had been disturbed during December, but there does not seem to have been an unbroken persistence of actually the same spot-group.

This northern group showed a strong tendency to extend itself in longitude. On March 4 it had an extreme length in longitude of $15\frac{1}{2}°$, on March 6 of 18°, its extreme breadth being about 7° of solar latitude. Up to the present only one photograph taken during the joint appearance of these two great spots has been measured and reduced. This was taken on March 4, when the areas and mean positions for the two groups were as follows :

Group, 1905 March 4.	Projected Area of		Area for Group corrected for Foreshortening		Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian.
	Umbra.	Whole Spot.	Umbra.	Whole Spot.			
					°	°	°
Southern Group	305	2358	159	1224	330·3	−16·7	+12·8
Northern Group	242	3133	189	2447	270·3	+10·6	−47·2

The following table gives the times when the Southern Group crossed the central meridian, and the angular distance between the centre of the group and the centre of the disc at the time of transit :

Times of Transit across the Central Meridian of the Southern Group.			Duration of Transit.	Distance of Centre from Centre of Disc.
Beginning.	Centre.	End.		
1905. d h	d h	d h	h	°
Jan. 8 3	Jan. 8 10	Jan. 8 17	14	−14·2
Feb. 3 14	Feb. 4 4	Feb. 4 18	28	− 9·3
Mar. 3 0	Mar. 3 11	Mar. 4 0	24	− 9·4

The successive times of transit of the centre of the group give the intervals $26^d\ 18^h$, and $27^d\ 7^h$ for the observed synodic rotation. The mean synodic rotation period for a spot in latitude $-15°$ is given by Carrington as $27^d\ 3^h$.

The Northern Group would appear to have been in transit about the following times :

Times of Transit across the C.M. of Northern Group.			Duration of Transit.	Distance of Centre from Centre of Disc.
Beginning.	Centre.	End.		
d h	d h	d h	h	°
March 7 8	March 8 0	March 8 17	33	+17·8

No really violent magnetic disturbance has taken place during the passage of these two great spots, but there have been several disturbances of moderate amount. The following table gives all the disturbances since 1905 January 1 in which the amplitude of the movement has exceeded 10′ in declination, or 60γ in horizontal force :

Period of Disturbance. Greenwich Civil Time.		Range in Declination.	Range in Horizontal Force (approximate).
From 1905. d h	To 1905. d h	′	
Jan. 5 17	Jan. 6 0	30	140γ
14 19	15 3	16	60
20 20	20 22	15	< 50
Feb. 1 21	Feb. 2 5	12	75
3 2	3 23	45	240
4 18	4 20	24	110
5 12	5 22	20	70
14 20	14 23	18	80
15 20	15 23	16	50
22 20	23 3	14	< 50
Mar. 2 13	Mar. 3 2	29	200

Royal Observatory, Greenwich :
1905 *March* 10.

Spectroscopic Observations of the Recent Great Sun-spot and Associated Prominences. By A. Fowler.

Introductory.

The great sun-spot which was so conspicuous during the last three days of January and the early part of February presented several features of interest when examined with the spectroscope, and it may be useful to give an account of the phenomena observed. Observations of the spectrum were made on January 31, February 1, 2, and 3, and after the return of the spot on February 25, 28, March 3 and 6. The prominences overlying the spot as it passed over the western limb on February 11 were also observed and the spectrum recorded in considerable detail.

All the observations were made with an Evershed solar spectroscope attached to a 6-inch refractor.

Reversals of Lines.

During the first passage of the spot across the disc the C and F lines of hydrogen were brightly reversed, but on its return the reversals were less pronounced and could not be seen at all on March 6. The D_3 line of helium was a prominent feature for a short time on February 2, and was also noted on February 25. Reversals of the sodium lines D_1 and D_2 were very conspicuous during the earlier stages, but were not seen after the return of the spot. The magnesium lines b_1, b_2, and b_4 and the iron line b_3 were observed to be reversed on January 31, and again, together with many other lines, on February 2. Details of the observations are as follows :

January 31 (9.50 A.M.–2.30 P.M.).—C and F were brilliantly reversed in numerous places over a large area, and especially over the greater part of the largest umbra. D_3 was suspected as a bright line. D_1 and D_2 were brightly reversed over the two largest umbræ, and the four *b* lines were seen bright on the inner edge of the principal umbra, where C and F were brightest. The displacements of the lines were very slight.

February 1 (11–11.20 A.M.).—C and F were reversed as on the previous day, but the displacement of the (dark) lines were more marked ; the greatest displacement was about 2 tenth metres. D_3 was not observed, but D_1 and D_2 were reversed as on January 31.

February 2 (9.45–12).—From 9.45 to 10 A.M. there was an eruption over the spot, during which the C line in a region preceding the principal umbra was expanded into a cloudy form extending about 5 tenth metres towards the red ; over the umbra the line was also brilliantly reversed, but occupied its normal position. D_3 was also very conspicuous, and, so far as

could be determined, its appearance was similar to that of the C line, except that it could not be traced quite so far from the normal position : it changed from bright to dark as the position of the spot upon the slit was varied.

During this eruption, at the place where C, F, and D_3 were most brilliant, all the Fraunhofer lines seemed to be either obliterated or reversed. There was only time to note the wave-lengths of a few of the bright lines but among them were certainly the helium line 6678·3, the D lines of sodium, the enhanced line of iron 5316·79 (1474 K), the iron line 5269·7 (E_2), the four *b* lines, and the enhanced iron lines 5018·6 and 4924·1 ; they were, in fact, among those most frequently observed in the spectra of metallic prominences on the Sun's limb. These bright lines did not appear to be in the least displaced from their normal positions.

At 10 A.M. the metallic lines had disappeared and the bright D_3 line over the umbra was greatly enfeebled, though it remained visible for some time as a faint dark line in the region preceding the umbra. At 11.10 A.M. D_3 was again seen as a bright line over one of the smaller umbræ at a place where the D lines were also reversed.

February 3 (9.50–12.30).—Though reversed in many places over the spot area, C and F were nowhere very bright, and D_3 was not detected. D_1 and D_2 were reversed over the largest umbra, where C and F were brightest.

February 25 (10.50–12.40).—After return of spot. C and F were still reversed over the largest umbra, and D_3 was also seen as a feeble bright line in the same region. D_1 and D_2 were much widened, but reversal was not certainly detected. Although the spot appeared to be well within the limb, the adjacent chromosphere was considerably disturbed, and several of the brighter chromospheric lines, including 5535·0, 5363·0, 5284·2, 5276·2, and 5197·8, were noted.

February 28 (9.50–12.30).—C and F were "knotted" and somewhat feebly reversed in several places, and, as before, they were brightest on the preceding edge of the largest umbra. D_3 was not detected, and no reversals of any metallic lines were observed.

March 3 (10–12).—The appearances were very similar to those of February 28.

March 6 (12.30–1).—The spot was now greatly reduced in size, and nothing abnormal about C, F, or D_3 was observed.

The phenomena which have been described seem to be sufficiently explained by the presence over the spot of prominences of varying activity, and it will be gathered that the greatest action was exhibited near the preceding edge of the largest umbra. Rapidly changing prominences were in fact observed on February 11, when the spot was on the Sun's limb.

As indicating that the appearance of D_3 is not restricted to very large spots, it may be mentioned that this line was very

prominent in the peculiar group of small spots seen in the eastern hemisphere on February 22; it then appeared as a broken and contorted dark line resembling C and F.

The Widened Lines.

General examinations of the spectrum of the largest umbra indicated that the "most widened" or most strengthened lines were not notably different from those recently observed in other spots. In other words, the majority of the affected lines could be traced to vanadium, titanium, chromium, scandium, sodium, calcium, and iron.

Selected parts of the spectrum were examined very minutely, the intensities of the lines being estimated by comparison with the Fraunhofer lines outside the spot spectrum. The result of these observations is to indicate that the spectrum, apart from the superficial effects already described, was sensibly constant from day to day, and was not materially different from that of another spot observed on the same plan on January 18. Thus, out of forty-six lines recorded in the great spot on January 31 in the region 5170–5239·2 forty-four appeared in the record for the corresponding part of the spectrum on February 1, and thirty-five of the forty lines recorded on February 28 in the region 5198·9–5239·2 appeared also in the previous lists. In the spot of January 18 forty affected lines were noted in the region 5188–5266, and thirty-seven of these occur among the lines recorded in the same part of the spectrum of the great spot. Considering the difficulty attending the observations, the differences are probably not significant.

The Spot Bands and General Absorption.

The bands of the sun-spot spectrum in the red, near 6381 and 6390, as well as those in the region more refrangible than *b*, were very strongly marked in the larger umbræ, and were always seen when looked for.

In the earlier observations the resolution of the general band of absorption, as described by Young* and Dunér,† was seen very clearly, although I had previously believed the dispersion at my disposal inadequate to show it. Probably the large dimensions of the umbra rendered the structure more visible. The resolution was best displayed by the bright gaps which occur here and there among the closely crowded dark components, and, as already noted by the observers named, these bright gaps are of the same brightness as the undimmed photospheric spectrum outside the spot. The general appearance of the band was very similar to that of a complex banded spectrum, such as that of sulphur, in which the maxima or "heads" are not very pronounced.

* *American Jour. Sci.* 3rd series, vol. xxvi. p. 333 (1883).
† *Recherches sur la Rotation du Soleil* (Upsala, 1891).

Under favourable conditions Young and Dunér were able to trace the dark components of the spot-band structure in the spectrum of the disc outside the spot, but this was not clearly seen in my observations; the same effect, however, is to some extent indicated by the fact that in some cases, the bright gaps occupy the spaces between nebulous lines of low intensity tabulated by Rowland. Three examples are as follows :—

Bright Gaps in Spot Band.	Solar Lines (Rowland).	
5160·8	5160·42	00 N
	5160·55	0000
	5161·01	0000 N
5201·6	5201·46	0000 N
	5201·77	0000 N
5259·4	5259·26	0000 N
	5259·66	000 N

It is accordingly not improbable that the absorbing vapour which is chiefly responsible for the darkness of a spot is thinly distributed over the general surface of the Sun, and may account for some of the very numerous faint lines of the Fraunhofer spectrum. At all events the appearance of the spot-band is in complete accordance with the view that the darkness of a spot is due to absorption and not to defective radiation from the stratum beneath the spot. Further, as pointed out by Young, the absorption must be produced by matter in the gaseous form, and not by solid or liquid particles.

Spectrum of Chromosphere and Prominences overlying Spot on Western Limb.

Magnificent prominences of rapidly changing form, varying in height from one to two minutes of arc, were observed on the morning of February 11, when the spot was passing over the western limb. The appearance mostly resembled one or more jets from a hose-pipe, and it was noted that in each case the streams were directed towards the equator.

The chromosphere was greatly disturbed to a considerable height, as indicated by the numerous long or "high-level" lines seen with a tangential slit, while still more intense action was evidenced by the appearance of many short metallic lines in the spectrum of the stump of one of the prominences.

The observing conditions were by no means good, but the positions of sixty-seven of the bright lines were ascertained by reference to Rowland's map. Seeing that so few observations of these bright lines, other than those of Young, have been published, it may be that details of the observations will be of value. In the following table the wave-lengths are indicated in the first column; in the second column the approximate intensities are

shown by the numbers, while "l" and "s" respectively indicate whether the lines were long or short; the third column shows the origins which seem to be most probable, taking account of intensities of the lines and recognising that enhanced lines are specially developed in the chromospheric spectrum; columns 4 and 5 indicate the frequency and brightness of the corresponding lines given by Young.* Following Sir Norman Lockyer, enhanced lines are indicated by the prefix "p," so that "p Fe" reads as "proto-iron," &c. These special lines have been chiefly identified by reference to the table of chromospheric lines given by Sir Norman Lockyer in the report on the eclipse of 1898 January 22.†

Wave-length.	Intensity and Character.	Probable Origin.	Young. Frequency.	Young. Brightness.	Remarks.
4861·53	100 l	H	100	80	F. Expanded on both sides.
4922·10	15 l	He	30	8	No attempt could be made to record all the bright lines in this region.
4924·11	25 l	p Fe	40	10	
5015·73	20 l	He	30	10	
5018·63	25 l	p Fe	30	15	
5167·50	20 l	Mg	20	10	b_4
5169·22	40 l	p Fe	40	25	b_3
5172·86	30 l	Mg	50	30	b_2
5183·79	40 l	Mg	50	35	b_1
5188·86	15 s	p Ti	10	5	Not seen separately
5189·02		Ca			
5197·8	25 l	...	10	10	
5200·4	10 s	...	2	4	Possibly Cr.
5202·52	10 s	Fe	4	3	
5204·68	15 s	Cr	4	5	Not seen separately.
5204·77		Fe			
5206·22	15 s	Cr, Ti	4	5	
5208·60	15 s	Cr	4	5	Both components were reversed.
5208·78	15 s	Fe			
5227·04	20 s	Fe, Cr	3	3	Both components were reversed.
5227·36	20 s	Fe			
5239·47	25 l	...	10	10	

* Scheiner's *Astronomical Spectroscopy*, pp. 423–426, and p. 184.
† *Phil. Trans.* vol. 197A, p. 151.

Wave-length.	Intensity and Character.	Probable Origin.	Young. Frequency.	Young. Brightness.	Remarks.
5265·0	15 l	...	3	3	
5269·72	30 s	Fe	10	2	E_2.
5270·44 5270·56	25 s	Ca Fe	5	2	E_1. Not seen separately.
5276·17 5276·24	35 l	p Fe Cr	10	10	Not seen separately.
5284·28	25 l	...	10	2–6	Fraunhofer line assigned to Ti by Rowland, but not by Hasselberg.
5316·79	45 l	p Fe	100	2–20	"1474 K."
5324·37	15 s	Fe	...	...	Not in Young's list.
5325·46	20 l	...	2	2	Fraunhofer line assigned to Co by Rowland.
5328·24	25 s	Fe	3	2	
5328·70 5328·75	15 s	Fe Fe	3	2	Not seen separately.
5340·12	10 s	Fe	...	...	Not in Young's list.
5341·21	10 s	Fe	2	1	
5363·06	30 l	...	20	5–10	
5365·07	10 s	Fe	1	1	
5367·67	10 s	Fe	1	1	
5371·66 5371·73	20 s	Cr Fe	10	3	Not seen separately.
5381·22	10 l	p Ti	3	2	Dark line attributed to Fe by Rowland
5383·58	10 s	Fe	...	...	Not in Young's list
5393·38	15 s	Fe	2	1	
5397·34	25 s	Fe	4	2	
5404·36	10 s	Fe	5	3	
5405·99	25 s	Fe	2	1	
5425·46	30 l	...	25	6	
5429·91	30 s	Fe	8	3	
5434·74	20 s	Fe	2	2	
5447·13	25 s	Fe	0	4	
5455·83	20 s	Fe	10	4	

Wave-length.	Intensity and Character.	Probable Origin.	Young. Frequency.	Young. Brightness.	Remarks.
5477·12	20 s	Ni	1	1	
5527·03	20 l	Sc	?40	?5	Probably Young's line 5525·9 (A°), which is wrongly corrected to 5528·64 (R) and attributed to Mg.
5535·06	35 l	...	50	12	Perhaps slightly less refrangible than 5535·0, but certainly not 5535·7 Ba; dark line attributed to Fe by Rowland.
					Definition poor from here, and eruption perhaps less active.
5875·87	95 l	He	100	90	D_3; very long and bright, and expanded in places.
5890·19	40 s-l	Na	25	2–10	D_2 A little longer than most of "short" lines, with brighter "hump."
5896·16	40 s-l	Na	25	2–10	D_1
5991·60	15 l	...	15	2– 5	
6141·94	20 s-l	Fe, Ba	15	3– 5	Moderate length, with brighter "hump."
6238·60	15 l	...	10	2	
6247·77	20 l	...	10	4	
6347·3	25 l	...	10	3	
6369·7	10 l	...	5	3	
6371·6	15 l	...	5	3	Dark line assigned to Fe by Rowland.
6432·9	25 l	...	10	2–10	
6456·60	20 l	p Fe	15	3–10	Author's identification.
6497·13	20 s-l	Ba, Fe	18	3–15	Moderate length, with hump.
6516·3	30 l	...	15	3–10	Perhaps a little more refrangible than dark line.
6563·05	100 l	H	100	100	C.
6678·3	35 l	He	25	10–50	
7065·5	35 l	He	100	5–20	

It will be seen that the above list is in close agreement with that of Young, so far as the two are comparable. In the region between b_4 and 5363, included in Young's revision, there is, in fact, only one line of intensity greater than 2 which does not appear in the table—namely, 5226·7 (5); and this may well have been missed in consequence of poor seeing when that region was under observation. In the red end Young's line 6191·8 is the

only line of greater intensity than 3 which I did not record. Three short iron lines of low intensity which I observed, probably in moments of good seeing, are not given by Young, but two of them occur in Lockyer's table of eclipse lines. The long line of scandium at 5527·03 is probably identical with a line observed by Young, which has been assumed to be the adjacent magnesium line 5528·64 when correcting from the scale of Ångström to that of Rowland. I have seen it on other occasions in metallic prominences, and am quite certain of its position. Notwithstanding the generally higher intensities which I have assigned to the shorter lines as compared with those given by Young, it would thus appear that the spectrum of the disturbed chromosphere is remarkably constant.

A comparison with the flash spectrum, as recorded by prismatic cameras, indicates that the long arcs of the eclipse spectra are represented by long lines in the foregoing table, suggesting that over a spot there is a general elevation of the chromospheric layers with little intermingling.

So far as it is possible to make a comparison of the bright chromospheric lines of February 11 with the affected dark lines previously noted in the spot, it appears that the high level lines were not among those intensified in the spot, while the common lines were chiefly those of iron, chromium, and calcium, which appear as strong Fraunhofer lines.

Addendum, 1905 *March* 13.

Brilliant prominences were again observed in the neighbourhood of the decaying spot when it was near the western limb on the morning of March 10. Many bright lines were seen, and the twenty-one identified were identical with long lines observed on February 11, and had the same relative intensities.

The Seasonal Variation in Magnetic Disturbance, with other Remarks. By William Ellis, F.R.S.

In my paper, *Monthly Notices*, 1899 December, I showed, from the records of the Royal Observatory for the fifty years 1848–97, that there existed greater frequency of magnetic disturbance at Greenwich at or near the equinoxes, and a lesser frequency at or near the solstices; and in again referring to this question, *Monthly Notices*, 1901 June, I said that "the spring maximum appears, on the whole, to fall somewhat before the equinox, and the autumn maximum somewhat after the equinox." I propose now, from the material contained in the first-mentioned paper, to endeavour to locate these points more precisely. At p. 153 of that paper the number of days of moderate, active, and great

disturbance occurring in the fifty years mentioned are separately given, arranged in half-monthly periods, these three sets of numbers being here combined to form the numbers in Table I. as given by observation; but as fifty years are apparently insufficient to produce uniformity, smoothed values have been added, found by twice taking in succession the means of three observed values.

Table I.

Number of Days of Magnetic Disturbance, 1848–97, in Half-monthly Periods.

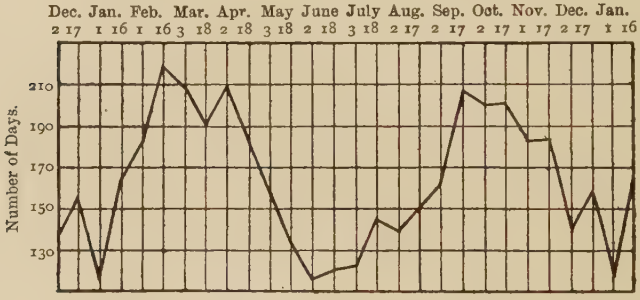
Middle Day of Half-monthly Period.	Total Number of Days of Moderate, Active, and Great Disturbance		Middle Day of Half-monthly Period.	Total Number of Days of Moderate, Active, and Great Disturbance	
	As Observed.	As Smoothed.		As Observed.	As Smoothed.
Jan. 1	116	146	July 3	121	127
16	165	163	18	145	136
Feb. 1	184	183	Aug. 2	140	143
16	219	200	17	149	156
Mar. 3	209	204	Sept. 2	162	171
18	189	200	17	209	190
Apr. 2	208	191	Oct. 2	202	197
18	178	176	17	203	197
May 3	155	156	Nov. 1	184	185
18	132	137	17	185	173
June 2	114	125	Dec. 2	139	155
18	119	123	17	156	148

The observed and smoothed values are graphically represented in the annexed diagram. Inspection of the smoothed numbers and smoothed curve alike shows that the points of maximum disturbance fall at about March 3 and October 9 respectively, the former in advance of the spring equinox by about the same number of days as the latter follows the autumn equinox. This close agreement in the interval of time—in the one case preceding the equinox by some seventeen days and in the other following the equinox by some seventeen days—may be, in a sense, accidental; and it will be interesting to see how far this result may afterwards be confirmed. As regards minima, that of summer appears to be much more pronounced than that of winter. The position of the points of maximum frequency of disturbance may possibly vary with the latitude of the place.

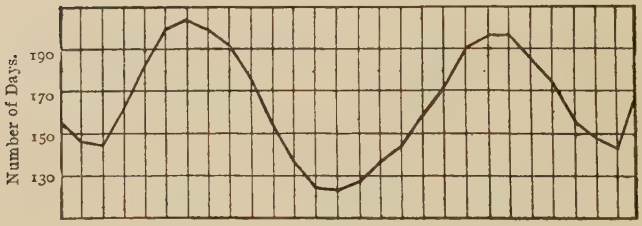
I have read with interest the paper by Mr. Maunder in which he endeavours to show the existence of direct relation between the rotation period of the Sun and the occurrence of terrestrial magnetic storms, and Professor Schuster's criticism thereof. Whatever may be the nature of any suggested explanation of their cause, it should satisfy established facts of observation. Perhaps I may be allowed to refer to some of

these. One circumstance is that active storms commence simultaneously over the whole earth—an accordance that has been shown to exist at stations widely separated both in latitude and longitude. The first impulse is usually of the nature of a sudden shock, more or less marked, in some cases being of extreme violence. But if the primary cause of magnetic storms be mainly of terrestrial origin, how is the undoubted general relation with sun-spot variation to be explained? Another matter is the already spoken of seasonal variation in the frequency of disturbance existing in our latitude, and by analogy the existence of similar seasonal variation (with maxima at the equinoxes) in southern latitudes, whatever may be the nature of the variation

Seasonal Variation of Magnetic Disturbance.



The above Curve smoothed.



in polar regions. The publication of the results of the reduction of the long series of Melbourne photographs, understood to have been taken in hand, might usefully confirm on this point what has been observed in our latitude.

A further circumstance which may be considered is that the first impulse in a magnetic storm, as well as at times being of considerable magnitude, has a tendency also to be of similar character in different storms, but having at different places its own distinctive character. In Table III., *Proc. Roy. Soc.* vol. lii. p. 205, an attempt was made to give particulars on this point in regard to seventeen separate storms observed at nine different places. The returns made to me were not complete, but at six

of the places the information is interesting. The particulars are contained in Table II., which indicates that at Greenwich, in

TABLE II

Direction of First Movement in Magnetic Storms.

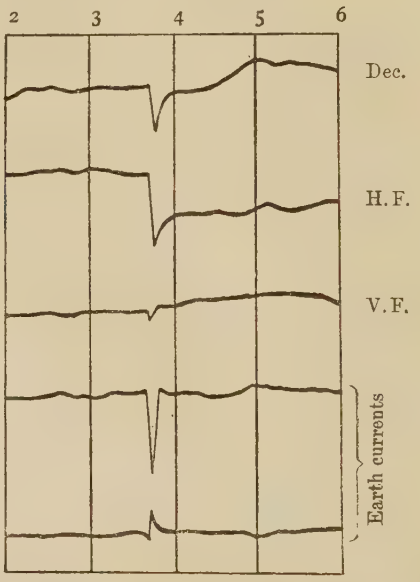
Place.	Number of Days on which the Returns were Complete.	Number of Days on which the First Movement was of Similar Character.	Direction of First Movement in Dec.	Direction of First Movement in Hor. Force.	Direction of First Movement in Vert. Force.
Greenwich	14	11	+	+	+
Pawlowsk	9	6	+	+	+
Mauritius	11	7	−	+	+
Bombay	17	17	+	+	−
Batavia	15	15	+	+	−
Zi-ka-wei	4	4	−	+	−

eleven out of fourteen cases, and at Pawlowsk, in six out of nine cases, the first movement was such as to increase all elements; that at Mauritius, in seven out of eleven cases, declination was decreased and horizontal and vertical force increased; and that at Bombay and Batavia declination and horizontal force were increased and vertical force decreased for the whole of the instances, seventeen and fifteen respectively, in which the returns were complete. At Zi-ka-wei, in addition to the four cases in which the information was complete, on eleven other days there was indication for either one or two of the magnets, all in harmony with the movements for Zi-ka-wei given in the table. At Melbourne there was no special distinctive movement. These results would appear to be of some importance as showing that at the commencement of a magnetic storm the first impulse is in general similar: that is, that the Earth becomes affected usually in one definite way. This seems rather to raise the question whether the first shock over the whole Earth occurs when any particular face of the Earth is turned to the Sun. As mentioned in my first quoted paper, p. 151, I once tabulated to a considerable extent these initial movements according to the hour of Greenwich time, and remarked (p. 152) that there was some reason to think that storms commence more frequently when the Earth occupied a given position; but the inequality was not very striking—certainly there was no part of the twenty-four hours at which these movements were either unusually numerous or very scarce.

Another matter is that earth-currents (the spontaneous currents that arise in telegraph wires), which at times of quiet magnetism are very weak, become when a magnetic storm arises powerful. The two phenomena are connected in the most striking manner. But a solitary magnetic movement, even if it be not great, only it be sudden or abrupt (the essential feature),

will be at once accompanied by an equally abrupt earth-current. At times isolated instances of this feature arise. An interesting illustration of a case of this kind, showing the close relation that exists on such occasions between magnetic and earth-current movements, is that occurring at Greenwich on 1893 February 14 at $3^h\ 45^m$ in the morning, when the isolated magnetic movements, though not large (in declination only four minutes of arc), were

Isolated magnetic and earth current movement recorded at Greenwich (copy of photograph).



accompanied by corresponding marked earth-current, the different elements all being in a quiescent state both before and after the little outbreak. It may be further noted that the diagram indicates increase of declination and of horizontal and vertical force, the movements being in this respect in harmony with the usual Greenwich direction of first movement in storms, as shown in Table II.

Provisional Elements of Jupiter's Satellite VI.

By A. C. D. Crommelin.

The information contained in *Publications of the Astronomical Society of the Pacific*, vol. xvii. No. 100, renders it tolerably certain that this object is really a satellite of *Jupiter*. Professor Aitken's visual observation with the 36-inch refractor proves that it is a real body in our system, not a mere series of

photographic defects; while the fact of its positions being in cacord with those of a satellite for a period of over eight weeks renders the hypothesis of its being a minor planet extremely improbable, though perhaps not absolutely impossible. Under these circumstances, and in view of the fact that the Lick observers are waiting for further observations before publishing definitive elements, it seems worth while to give a rough approximation to the orbit, which I have deduced from the material already available: this is quite insufficient to deduce the eccentricity, so that the orbit is necessarily assumed to be circular.

The following table contains all the available material:

1904 December 3, 8, 9, 10.—Satellite west of *Jupiter* and receding from it.

1904 December 25.—West elongation, distance about 50′.

1905 January 4·7 G.M.T.—Distance 45′, position-angle 269°. Approaching *Jupiter* 45″ daily.

1905 January 17·7 G.M.T.—Distance 36′, position-angle 266°.

1905 January 28.—Approaching *Jupiter* about 1′ daily.

Taking first the elongation distance as exactly 50′, I examined how nearly this would represent the positions on January 4 and 17. It is necessary to do this independently on the two hypotheses of direct and retrograde orbital motion, since we cannot as yet distinguish between these. I may remark that the Lick astronomers have now definitely stated that the phrase "apparent motion retrograde" in the original telegram had reference only to the diminishing position-angle; it was fairly obvious from the nature of the case that this must be so, though several astronomers interpreted it as referring to the orbital motion. It may perhaps be suggested that the words "position-angle diminishing" should be used in similar cases in the future to avoid all ambiguity. An elongation distance of 50′ on 1904 December 25 implies that the distance of the satellite from *Jupiter* is 0·0668 in astronomical units, or about 6,200,000 miles, the corresponding sidereal period being 204 days.

From these data the computed angular distances from *Jupiter* are as follows:

G.M.T.	Computed Distance.		Observed Distance.
	Direct Orbit.	Retrograde Orbit.	
	′	′	′
1905 January 4·7	45·8	46·0	45
17·7	35·4	34·3	36

It will be seen that the results are slightly more accordant on the "direct" hypothesis; but no stress can be laid on this, since the discordances may be due to eccentricity in the orbit or an error in the assumed epoch of elongation. It appears probable that the assumed distance of the satellite at elongation is correct within one or two per cent., and the observations do not permit of a more precise determination.

The calculated rate of approach to *Jupiter* is 42″ daily on January 4, 72″ on January 28, these values being tolerably accordant with the observed values given above.

To find the approximate position-angle of the apse it is sufficient to note that the linear velocity parallel to the minor axis of the apparent ellipse, being a maximum at the apse, is nearly uniform for some days after this. This method gave 270°·7 as the position-angle, which a second approximation altered to 270°·9. The deduced minor semi-axis of the apparent ellipse on December 25 is 4′·96.

If ϕ be the angle between the line of sight on December 25 and the orbit plane, then $\sin\phi = \frac{4\cdot96}{50}$, and $\phi = 5°\cdot7$.

Then to find the pole of the orbit plane we must proceed along the minor axis of the apparent ellipse (*i.e.* in position-angle 0°·9) to a distance of 84°·3 on the hypothesis of direct orbital motion, or 95°·7 on the retrograde hypothesis. The two points thus indicated were marked on a large scale map, and their distances from the poles of *Jupiter's* equator and orbit were measured with the following results :

	Direct Hypothesis.	Retrograde Hypothesis.
Inclination of satellite's orbit to plane of *Jupiter's* equator	26°·0	24°·7
„ „ „ „ „ orbit	23·8	23·9

It may be noted that the position of the orbit plane would not be greatly altered by a moderate eccentricity of the orbit. We may, in fact, regard it as certain that, if the object be a satellite of *Jupiter* at all, its orbit plane is inclined at least 20° to both the equator and the orbit of its primary. This large inclination is very surprising and perplexing. We have a partial analogy in the case of *Japetus* and *Phœbe*; but these deviate from the primary's equatorial plane towards its orbital plane, whereas the present body makes a large angle with both. The seventh satellite, judging from the meagre reports received, is inclined at a still higher angle and in a totally different direction. These new satellites are indeed impairing the symmetry of the solar system by the peculiarities of their orbits.

It is noteworthy that according to the direct hypothesis the pole of the orbit plane is only 1°·5 from our own north pole, so that in this case the major axis of the apparent ellipse will always point very nearly east and west. The satellite will again be in west elongation next July, and a determination of its position-angle then will probably be sufficient to decide the direction of orbital motion.

Unfortunately the numeration of *Jupiter's* satellites is now in precisely the same confusion as that of *Saturn's* system was before the numbers were abandoned and names substituted. A similar course would seem to be advisable here ; the designation V. for the inner satellite was tolerated for a time, as it was

considered to be in a class by itself; but it has now got companions, so that this subterfuge disappears. The substitution of names for numerals is certainly more poetic, and abbreviations may be devised which would take no more space in printing than the present notation (e.g. *Io.*, *Eu.*, *Gan.*, *Cal.* for the four old satellites).

It may help to realise the relative distances of satellites from their primaries to point out that the distances of satellites V. and VI. from *Jupiter* are comparable with those of *Mercury* and *Uranus* from the Sun, while those of *Mimas* and *Phœbe* are comparable with those of *Mercury* and *Neptune*.

Benvenue, 55 *Ulandi Road*, *Blackheath*, *S.E.*:
1905 *March* 8.

The Later Leonids of 1904 *November.*

By Rev. S. J. Johnson, M.A.

As most of the observations obtained of this shower relate to what was seen in the earlier hours of the morning of November 15 last, they may perhaps be supplemented by some notes on the meteors nearer sunrise, or between 3.30 and 5.30 A.M. A perfectly clear sky and the absence of even the crescent Moon of 1903 favoured the display. During the two hours aforesaid I noticed twenty-five meteors (not quite all Leonids) between 3.30 and 4.30, and thirteen between 4.30 and 5.30; but from the circumscribed portion of the heavens presented to the observer through obstructions and the delay occasioned by recording the tracks it would be probably correct to multiply this number by 4. This would make just 100 meteors in the two hours. Comparing with 1903, when I noticed fifty-three in $1\frac{3}{4}$ hour, equivalent to about sixty in two hours, and multiplying again by 4, we obtain 240 for the same hours in the morning in 1903. This makes the stream of 1903 two and a half times as plentiful as that of 1904. A noteworthy point was the intense green colour of the larger ones, probably magnesium. Eleven were = ordinary 1st-magnitude stars, one = *Vega*, two = ♃, and one = ♀. The three brightest meteors seen were $14^d\ 16^h\ 10^m$ = ♃, across *Com. Beren.* 175°+28° to 198°+28°, $14^d\ 16^h\ 14^m$ = ♀, very green. From about 195°+22° to about 202°+18°, $14^d\ 17^h\ 16\frac{1}{4}^m$ = ♃, across χ *Leonis*, almost to *Mars*, 155°+19° to 167°+5°.

Melplash Vicarage, *Bridport*:
March 4.

The Great Nebula of ψ Eridani. By Dr. Max Wolf, Assoc. R.A.S.

I have had much difficulty in securing a photograph of this large nebula. It is situated somewhat too far south for our latitude, and between very bright stars, so that the sky here has never yet been sufficiently clear for me to obtain a perfect image. I do not know if the nebula is known elsewhere—it is not in the N.G.C. nor in the Index Catalogue. The small nebulæ 398 and 402 in the Index Catalogue seem to be included in this enormous nebula, as well as N.G.C. 1779, 1797, 1799, but they are all relatively small and difficult objects. Perhaps Professor Barnard has also photographed this object.

The nebula now reproduced is situated between λ *Orionis* and ψ *Eridani*, not far from the first-magnitude star β *Orionis*.

The first traces of this nebula were found on a plate taken on 1891 January 16 with a 4-inch Millet portrait lens and with 1^h exposure. It appears on many plates, including those of 1894 November 24 (in the centre of the plate); 1896 February 3, 7; 1899 February 3, 4, 10; 1900 March 1, &c., till 1904 November 16—all taken with smaller lenses up to 6 inches.

The accompanying plate (15, Fig. 1) is from a photograph taken with the 16-inch Brashear lens on 1905 January 8, with $4^h\ 21^m$ exposure. It was not wholly satisfactory on account of haze, to which is due the large halos round the bright stars. But the weather has since been too unfavourable for me to repeat the exposure, and now it is too late in the spring for this latitude.

The bright star on the right is ψ *Eridani*. The denser parts of the nebula extend along the boundaries of *Orion* and *Eridanus*. There is no doubt that it is attached to the *Orion* nebulæ, all plates showing numerous traces of connexion. The nebula is composed of several streams of nebulosity, and has the appearance of foggy clouds driven by a fresh breeze. It belongs to the same class as Barnard's great nebula near ξ *Persei*, and in many features these two objects singularly agree. The scale of the reproduction is about 4 cm. to 1°.

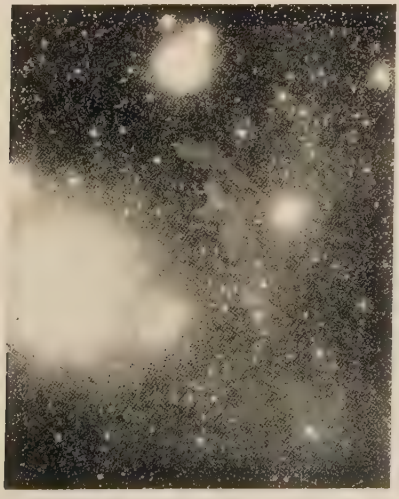
To illustrate its position in reference to the *Orion* nebulæ I give (plate 16) a sketch of the southern part of *Orion*, in which I have outlined from my plates the principal nebulous branches.

A is the great *Orion* nebula, B its well-known northern branch around *c Orionis*, C is the beautiful nebula following ζ *Orionis*, D its southern branch, &c. K, L, M, O, Q are especially bright clouds of the large spiral; E is a denser stream near ε *Orionis*; the exceedingly interesting wisps U, V, W are placed around and west of η *Orionis*; T and Q_1 are especially strong clouds. Dense diffused nebulosity extends for some distance round S. The central part of southern *Orion* between P, T, R, E is relatively without nebulosity.

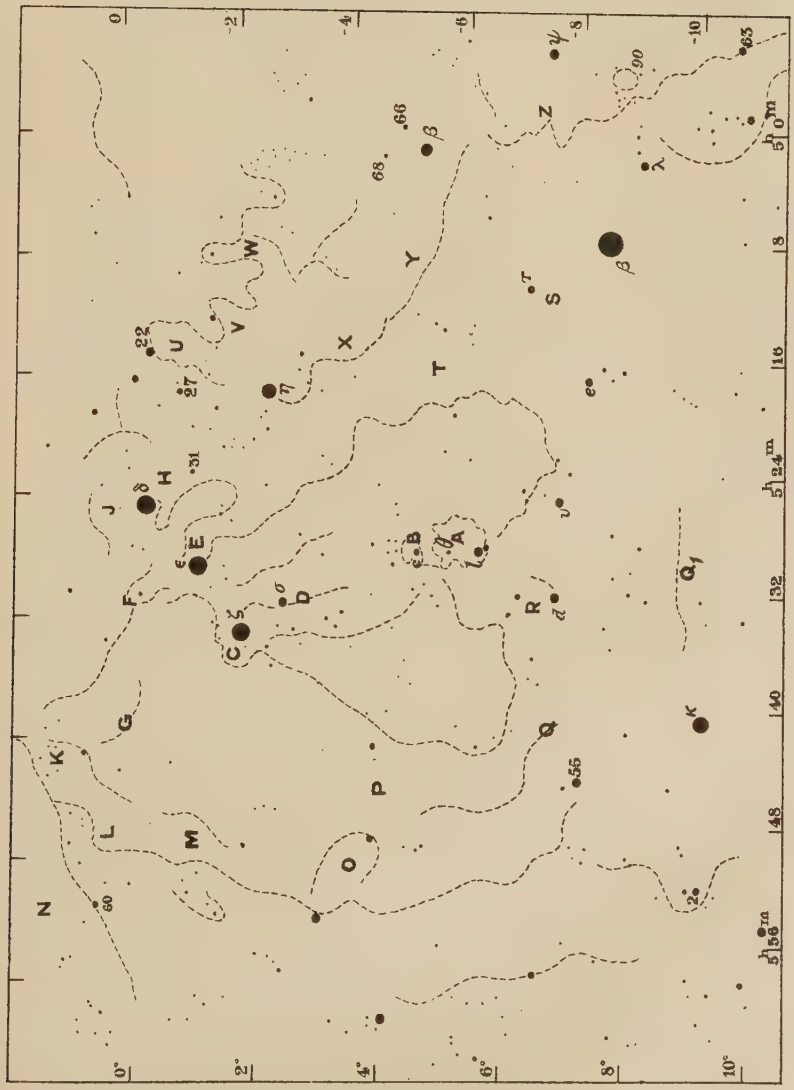
The ψ *Eridani* nebula is marked Z on the chart. It is not far from β *Orionis*, and nearly midway between λ *Orionis* and

FIG. 1.

FIG. 2.



GREAT NEBULA OF ψ ERIDANI. DR. MAX WOLF, HEIDELBERG.



SKETCH MAP OF NEBULÆ IN SOUTHERN PART OF ORION.

ψ Eridani, extending from S.S.W. to N.N.E. The clouds become dense near 63 *Eridani*, and reach to about 1° south of *β Eridani*. I give some coordinates taken from the *B.D. Atlas* :—

R.A. h m	N.P.D. ° ′	
4 53·3	100 17	Cloud.
56·7	98 37	Elliptical cloud.
59·5	97 23	Fan-shaped end near the middle of the plate.
57·0	96 53	Long wisp.
58·9	97 14	Cloud.
59·0	97 0	„
5 0·8	96 30	Fan-shaped cloud.
0·3	96 20	„ „

The star-like spot $5^h\ 0^m{\cdot}3$, 96° 40′ seems on comparison with other plates not to be real.

To show more clearly the position of this nebula I give on plate 15, Fig. 2, a portion of a photograph taken with the 6-inch lens on 1904 November 16 by my assistant, Mr. Goetz. This plate was only exposed for $1^h\ 20^m$, yet by suitable printing the features of the nebula can be brought out without too much halo round the bright stars.

The large white spot on the left of the plate is the glare from *β Orionis* ; at the top is *β Eridani*, at right of the centre *ψ Eridani*. We here see very well between the bright stars the long track of the nebula extending from *β Eridani* to 63 *Eridani*. This reproduction shows that the stream of nebulosity forms in its northern parts the brighter boundary of an extended nebulous region involving *ψ Eridani*. The southern portion spreads around 63 *Eridani*, and is divided by complicated channels. The denser spot north-east near the star 90 of the chart is on all plates taken with the Voigtländer lens, but not well shown on those taken with the 16-inch—perhaps because too near the edge of the plate.

It is seen that nebulous patches are spread over several parts of the plate, forming connexions with the S and Y districts of the *Orion* system. The scale of the plate is about 9 cm. to 1°.

The intensity of the Z nebulæ is relatively great. They come out with less than an hour's exposure. The brighter parts east of *ψ Eridani* are almost equal in intensity to the D nebulæ in their parts east of *σ Orionis*.

The *ψ Eridani* nebula would be a beautiful object for a reflector, especially in more southern latitudes.

Königstuhl, Astrophysical Observatory :
1905 *February* 20.

Note on the Publication of Astronomical Papers, with special reference to the International Catalogue. By W. W. Bryant.

It has been suggested that in view of the approaching Conference on the International Catalogue of Scientific Literature which was to take place when the scheme had been working for five years a few notes on the practical working of it would be very useful to the Regional Bureau, which in the case of astronomical papers published in the United Kingdom is the Royal Astronomical Society, represented by a committee, and that the best form for these notes would be a short paper presented to the Society.

I undertook this the more readily as I have had to deal with the working of the schedule and instructions, not only for the United Kingdom, but also indirectly for the whole world, as I have revised all the slips sent in for the three volumes already issued, and have also had the opportunity of testing the application of the same schedule to a great deal of literature already published before the commencement of the International Catalogue.

The most important point on which I desire to lay stress is the increasing need for centralisation of special scientific literature in special publications. It may seem that in the case of astronomy in this country the list of publications in which occasional contributions of scientific value appear is not a long one, but astronomy is only one of seventeen sections, and some of these sections are so much overweighted that the expenses connected with their issues are a great stumbling-block to the success of the scheme, which must ultimately stand or fall by the financial support given to it. And a rule limiting the quantity of any one science is hardly admissible; I think, therefore, we should press for a general rule, on the lines of the following suggestion.

Certain periodicals to be adopted for each science as "standard." Any astronomical articles appearing in them to be indexed under Schedule E. Papers of astronomical value appearing elsewhere to be, as far as possible, reprinted in one or other of the "standard" periodicals.

I am aware that this proposal has obvious disadvantages. It appears to discourage the publication of astronomical articles in periodicals of general interest, and thus to prevent the appeal of the science to a larger public. It lays the burden of deciding what is worth reprinting on some person or persons in what might appear a somewhat invidious manner, and the person or persons considered competent might be unable or unwilling to undertake the responsibility. Leaving the other objections to be raised elsewhere I will say a few words on these two.

As to the first, I take it the intention of the Catalogue is to enable scientists to trace work done in any special section of

their special science, and to them articles meant to popularise science with the general reader do not appeal. They have their publishers and their public, and will not suffer appreciably. Books, of course, are few in number, and will not be affected by the proposal. As regards another class of publication, original observations, often of value, are made by people who desire prompt and unquestioning publication. Their purpose being thus served what is to prevent them from sending in complete series to our Society or elsewhere if they desire scientific recognition? Some of them already do this, and in one department, as is well known, one of our Fellows, Mr. Denning, to a great extent does it for them. And this brings me to the partial disposal of the second objection mentioned above. I think the scientific editors of the "standard" periodicals (or the secretaries and council of the Societies in the case of publications by Societies) would find practically no difficulty in carrying out my suggestion, if the original authors will go so far as to submit their papers and observations.

One more word as to accessibility. It has been necessary in the Royal Society Catalogue work to send a small staff of indexers and assistants to the British Museum and the Natural History Museum to get at some publications not to be found at Burlington House. This not only involves a great amount of time, but invites the question as to whether it is worth while indexing papers so difficult to unearth. The difficulty was probably not foreseen at the time of publication, but there is no excuse for not guarding against a repetition of it in the future.

Royal Observatory, Greenwich:
1905 *March.*

Observations of Uranus at Windsor, New South Wales. By John Tebbutt.

As the planet *Uranus* during July and August last passed very close to the well-determined star No. 4648 of the Radcliffe Catalogue for 1890, the opportunity was taken for comparing the two objects by means of the filar micrometer of the 8-inch equatorial. The accompanying table gives the results of the measures. The planet was at no time sufficiently well defined to admit of the limbs being observed, so that the centre of the disc was chosen as the point for observation. The resulting corrections to the Mean Noon Ephemeris of the *Nautical Almanac* have been kindly supplied by Mr. Merfield, and are given in the last column of the table.

Observations of Uranus.

1904.	Windsor Mean Time.	Planet—Star.		No. of Comps.	Reductions to App. Place.		Parallax Corrections.		Concluded Geocentric Apparent Places of Planet.		Obs.—Cal.	
		$\Delta\alpha$.	$\Delta\delta$.		α.	δ.	α.	δ.	α.	δ.	α.	δ.
	h m s	m s	′ ″		s	″	s	″	h m s	° ′ ″	s	″
July 18	8 14 18	+0 26·12	+1 51·1	10	+3·19	+8·4	−0·01	−0·1	17 46 17·08	−23 37 3·9	0·00	−0·1
„ 19	7 37 28	+0 17·49	+1 55·1	12	+3·19	+8·4	−0·02	−0·1	17 46 8·44	−23 36 59·9	−0·01	+1·3
„ 26	7 0 22	−0 40·67	+2 13·0	12	+3·16	+8·3	−0·02	−0·1	17 45 10·25	−23 36 42·1	+0·03	+0·6
„ 29	6 35 41	−1 3·47	+2 20·4	15	+3·15	+8·2	−0·02	−0·1	17 44 47·44	−23 36 34·8	+0·02	+0·2
Aug. 1	6 46 2	−1 25·00	+2 27·8	15	+3·13	+8·2	−0·02	−0·1	17 44 25·89	−23 36 27·4	+0·05	+0·2
„ 2	6 34 36	−1 31·84	+2 29·6	12	+3·12	+8·2	−0·02	−0·1	17 44 19·04	−23 36 25·6	−0·01	−0·4
„ 3	6 30 45	−1 38·47	+2 32·8	11	+3·11	+8·2	−0·02	−0·1	17 44 12·40	−23 36 22·4	+0·02	+0·4
„ 5	6 37 18	−1 51·47	+2 37·0	8	+3·10	+8·1	−0·02	−0·1	17 43 59·39	−23 36 18·3	−0·09	−0·2

Mean Place of Comparison Star for 1904·0.

α.	δ.	Authorities.
$17^h\ 45^m\ 47^s{\cdot}78$	$-23°\ 39'\ 3''{\cdot}3$	Argent. Gen. Cat. 1875. No. 24243. Radcliffe Cat. 1890. No. 4648.

Observatory, Peninsula, Windsor, N.S. Wales:
1905 *January* 23.

Observations of Uranus and Saturn taken with the 6-inch Telescope of the Transit Circle of the Sydney Observatory.
By C. J. Merfield.

The accompanying observations of *Uranus* and *Saturn* were obtained with the telescope attached to the circles of the meridian instrument of the Sydney Observatory.

The writer, who is at present under the direction of Mr. Lenehan, acting Government Astronomer, was able to secure these observations during the evening's work with this instrument.

The equatorial instrument of this observatory requires a thorough overhauling, which will be undertaken at an early date; when completed it is the intention to secure observations of minor planets, comets, and southern double stars. The present writer anticipates being able to undertake this work.

URANUS.

1904.	M.T. Sydney.	$\Delta\alpha$.	$\Delta\delta$.	Cp.	α app.	δ app.	Log $p\Delta s$.	Star reductions to apparent.	Star.
	h m s	m s	′ ″		h m s	° ′ ″		s ″	
July 16	10 7 41	+8 4·99	+0 48″·7	7 1	17 46 34·53	−23 37 9″·8	0·1863$_n$	+3·18 +7″·9	1
16	10 7 41	+0 43·56	...	7	17 46 34·56	...	...	3·19 ...	4
18	9 59 31	+0 25·51	+1 50·7	7 1	17 46 16·51	−23 37 4·3	0·1864$_n$	3·19 8·4	4
26	9 26 57	+6 39·78	+1 16·7	7 1	17 45 9·28	−23 36 42·0	0·1867$_n$	3·14 7·7	1
26	9 26 57	+4 15·88	...	7	17 45 9·32	...	...	3·15 ...	3
27	9 22 53	+6 32·20	...	7	17 45 1·70	...	...	3·14 ...	1
27	9 22 53	+4 8·32	...	7	17 45 1·76	...	...	3·15 ...	3
29	9 14 46	+6 17·40	+1 24·5	7 1	17 44 46·89	−23 36 34·3	0·1868$_n$	3·13 7·6	1
29	9 14 46	−1 4·13	...	7	17 44 46·83	...	...	3·15 ...	4
Aug. 2	8 58 34	+5 48·90	+1 33·7	7 1	17 44 18·36	−23 36 25·1	0·1869$_n$	3·10 +7·6	1
2	8 58 34	+3 24·86	...	7	17 44 18·26	...	...	+3·11 ...	3

1904	M.T. Sydney.	$\Delta\alpha$.	$\Delta\delta$.	Op.	α app.	δ app.	Log $p\Delta\delta$.	Star reductions to apparent.	Star.
	h m s	m s			h m s	° ′ ″		s ″	
Aug. 2	8 58 34	−1 32·55	...	7	17 44 18·38	...	...	+3·12 ...	4
3	8 54 32	+5 42·26	+1 36·8	7 1	17 44 11·71	−23 36 22·0	$0\cdot1869_n$	3·09 +7·6	1
3	8 54 32	+3 18·34	...	7	17 44 11·73	...	...	3·10 ...	3
3	8 54 32	−1 39·21	...	7	17 44 11·71	...	...	3·11 ...	4
5	8 46 27	+5 29·50	+1 40·8	7 1	17 43 58·93	−23 36 18·1	$0\cdot1869_n$	3·07 7·5	1
5	8 46 27	+3 5·44	...	7	17 43 58·80	...	...	3·07 ...	3
5	8 46 27	−1 51·89	...	7	17 43 59·02	...	...	3·10 ...	4
10	8 26 19	+5 0·50	+1 53·0	7 1	17 43 29·88	−23 36 6·0	$0\cdot1871_n$	3·02 7·4	1
10	8 26 19	+2 36·86	...	7	17 43 30·18	...	...	3·03 ...	3
10	8 26 19	−2 20·78	...	7	17 43 30·08	...	...	3·05 ...	4
13	8 14 16	+4 45·80	+1 57·5	7 1	17 43 15·14	−23 36 1·5	$0\cdot1871_n$	2·98 7·4	1
13	8 14 16	+3 32·65	...	7	(17 43 15·05)	...	...	2·98 ...	(2)
13	8 14 16	+2 21·69	...	7	17 43 14·97	...	...	2·99 ...	3
13	8 14 16	−2 35·80	...	7	17 43 15·02	...	...	3·01 ...	4
16	8 2 15	+4 32·13	+2 3·4	7 1	17 43 1·43	−23 35 55·6	$0\cdot1872_n$	2·94 7·4	1
16	8 2 15	+2 8·05	...	7	17 43 1·39	...	...	2·95 ...	3
16	8 2 15	−2 49·25	...	7	17 43 1·53	...	...	2·97 ...	4
17	7 58 15	+4 28·13	+2 5·2	7 1	17 42 57·42	−23 35 53·8	$0\cdot1872_n$	2·93 +7·4	1
17	7 58 15	+3 15·14	...	7	(17 42 57·49)	...	...	2·93 ...	(2)
17	7 58 15	+2 4·16	...	7	17 42 57·39	...	...	2·94 ...	3
17	7 58 15	−2 53·21	...	7	17 42 57·56	...	...	+2·96 ...	4

1904	M.T. Sydney.	Δα.	Δδ.	Cp.	α app.	δ app.	Log $p\Delta_\delta$.	Star reductions to apparent		Star.
	h m s	m s	′ ″		h m s	° ′ ″		s	″	
Aug. 20	7 46 16	+1 53·27	...	7	17 42 46·46	...	...	+2·90	...	3
20	7 46 16	−3 4·25	+3 4·9	7 1	17 42 46·48	−23 35 50·7	$0·1873_n$	2·92	+7·8	4
22	7 38 19	+4 11·93	+2 11·1	7 1	17 42 41·15	−23 35 48·0	$0·1873_n$	2·86	7·3	1
22	7 38 19	+1 47·93	...	7	17 42 41·09	...	...	2·87	...	3
23	7 34 19	−3 13·25	+3 9·0	7 1	17 42 37·43	−23 35 46·6	$0·1873_n$	+2·87	+7·8	4

Mean Places of Comparison Stars.

Star.	α 1904.	δ 1904.	Authorities.
	h m s	° ′ ″	
1	17 38 26·36	−23 38 6·4	Cape 1850; Yarnall 1860; Argent. Gen. Cat. 1875; Paris 1875; Cape 1880.
2	17 39 39·42	...	Cape D.M. 23°, 6642.
3	17 40 50·29	−23 38 46·8	Yarnall 1860; Argent. Gen. Cat. 1875.
4	17 45 47·81	−23 39 3·4	Madras 1835; Argl. 1850; Argent. Gen. Cat. 1875; Cord. Z.C. 1875; Paris 1875; Radcliffe 1890.

SATURN.

1904.	M.T. Sydney.	Δα.	Δδ.	Cp.	α app.	δ app.	Log $p\Delta_\delta$.	Star reductions to apparent.		Star.
	h m s	m s	′ ″		h m s	° ′ ″		s	″	
Oct. 3	8 22 46	+8 56·25	+0 21·9	7 1	21 9 32·24	−17 36 12·5	$0·3864_n$	+2·86	+18·3	1
24	6 59 25	+8 10·09	−2 10·3	7 1	21 8 45·75	−17 38 45·9	$0·3853_n$	+2·53	+17·1	1
25	6 55 32	+8 12·51	−1 57·4	7 1	21 8 48·15	−17 38 33·1	$0·3854_n$	+2·51	+17·0	1
31	6 32 18	+8 34·49	−0 3·6	7 1	21 9 10·03	−17 36 39·6	$0·3862_n$	+2·41	+16·7	1

Mean Place of Comparison Star.

Star.	α 1904.	δ 1904.	Authority.
	h m s	° ′ ″	
1	21 0 33·13	−17 36 52·7	*Nautical Almanac*, 1904 (θ Capricorni). The correction +0ˢ·019 has been applied to α 1904. *See* Clock Star List 1904, Royal Observatory, Greenwich.

Comparison of Observations with the Mean Time Ephemerides of the Nautical Almanac.

URANUS.

Date 1904.	$d\alpha$ s	$d\delta$ "	Date 1904.	$d\alpha$ s	$d\delta$ "
July 16	+0·10	−1·0	Aug. 10	+0·04	+1·0
8	+0·07	−0·8	13	+0·15	−0·4
26	−0·13	+0·4	16	−0·08	+0·2
27	+0·05	...	17	0·00	+0·4
29	+0·24	+0·4	20	−0·05	−0·9
Aug. 2	−0·07	−0·3	22	+0·90	−0·8
3	−0·01	+0·5	23	+0·04	−0·7
5	0·00	−0·3			

SATURN.

Date 1904.	$d\alpha$	$d\delta$
Oct. 3	−0·09	+0·7
24	−0·12	+0·3
25	+0·06	+0·6
31	+0·07	+1·3

Erratum in Annual Report (*Obituary of Andrew Henderson*).

Page 337, line 12 from bottom, *for* Dundee *read* Paisley.